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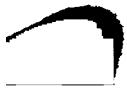
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KEY

TO

OLNEY'S

PRACTICAL ARITHMETIC,

WITH

HINTS TO TEACHERS,

AND

FORMS OF SOLUTIONS.

By EDWARD OLNEY,

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF MICHIGAN, AND AUTHOR OF A SERIES OF MATHEMATICAL TEXT-BOOKS.

NEW YORK:
SHELDON AND COMPANY,
8 MURRAY STREET.
1860.

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118.80.645

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PREFATORY NOTE.

THROUGH the earlier pages of this KEY, where there can be little need of teaching any one *how* to solve the examples, the chief attention has been given to hints as to methods of teaching and to forms of work. Great pains should be taken by the teacher that the pupil cultivate taste and ability for neat and elegant work on blackboard and slate. Considerable attention has been given to forms of solution and methods of explanation through all the earlier part of the book. In the latter part, little but solutions, with occasional examples of forms, has been attempted. It is thought that the KEY is sufficiently full, so that no teacher at all fitted for his work will be in the least embarrassed in the use of the book.

AUTHOR.

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KEY

TO

PRACTICAL ARITHMETIC.

ADDITION.

Pages 28-31.

THESE examples are designed simply to give pupils *facility in explaining the process of addition.* They are not a tithe of what will be needed to make pupils expert and accurate in adding. A good-sized volume would be required to contain all the drill-exercises necessary to give facility in the practical process. For this, oral exercises, charts, figures on the blackboard, every possible expedient to supply drill, will be needful. See ARITHMETICAL EXERCISES, pp. 6–12.

As a specimen of the manner of using these examples, we give the following: —

Example I.

EXPLANATION AS GIVEN BY THE PUPIL STANDING AT THE BOARD AND TRACING THE WORK WITH A POINTER, *his name being written above his work.* — "Add 427, 342, 856, and 728. I write the numbers as on the board, so that all the units shall fall in the right-hand column, all the tens in the next, and the hundreds in the next, in order that I may the more readily ascertain how many units there are in all of the numbers, then how many tens, and then how many hundreds.

WORK AS PUT ON THE BLACKBOARD. (*Pupil's Name.*)

427
342
856
728
2353

"I *begin to add* with the units column, in order that if any tens arise from adding this column I may add them in with the next column as I proceed. Adding, I say 8, 14, 16, 23; and, finding that there are 23 units in this column, I consider that 23 units is 2 tens and 3 units. Hence I write the 3 units under units column, and, proceeding to the tens column, I add the 2 tens in with it; thus, 2, 4, 9, 13, 15. Finding that there are 15 tens, I consider that 15 tens makes 1 hundred and 5 tens. Hence I write the 5 tens under tens column, and, proceeding to the hundreds column, add the 1 hundred in with it; thus, 1, 8, 16, 19, 23. Finding that there are 23 hundreds, I consider that 23 hundreds are 2 thousands and 3 hundreds. Hence I write the 3 in hundreds place, and the 2 in thousands.

"Thus I have added 427, 342, 856, and 728, since I have found one number 2353, called their sum, which is made up of all the others put together." Then the pupil should repeat definitions **38** and **32.**

In like manner each pupil should be trained to give, in good language, an explanation of every solution.

Pages 32-40.

The explanation of these solutions to be given by the pupil at the board will consist in *giving the reason for adding*, and not in giving an explanation of the *process* of adding. Thus Example 2, which will be solved *mentally*, with *a large* number of others given impromptu by the teacher, will be solved thus: The pupil rises, and says, "If James rode 8 miles and walked 3, he travelled the sum of 8 and 3 miles, which is 11 miles."

See examples of oral solutions in text-book, and also find additional exercises in the ARITHMETICAL EXERCISES, pp. 12–17.

Example 31, Page 35.

PUPIL'S EXPLANATION. — "If a merchant bought cloth for $375, and gained $50 in selling it, he sold the cloth for $375 + $50. In like manner, if he bought silk for $95, and gained $45 in selling it, he sold the silk for $95 + $45. Hence, in all, he sold cloth and silk for $375 + $50 + $95 + $45, or $565."

FOR BLACKBOARD. (*Pupil's Name.*)

375
50
95
45
565

Example 33, Page 35.

PUPIL'S EXPLANATION. — "From the time the compass was discovered to the birth of Christ was 1120 years; from the birth of Christ to the discovery of America was 1492 years; and from the discovery of America to the application of steam to propelling boats was 315 years. Hence, from the time of the invention of the compass to the time of the application of steam to the propulsion of boats was 1120 + 1492 + 315, or 2927 years."

FOR BLACKBOARD. (*Pupil's Name.*)

1120
1492
315
2927

MAKING CHANGE.

Page 39.

2. To the 25¢ I first add 5¢ by laying it out, and say "30." This means that with the 25¢ this 5¢ makes 30¢. Then I lay out 10¢, and say "40;" then another 10¢, and say "50;" then another, and say "60;" then another, and say "70;" then another, and say "80;" then another, and say "90;" then another, and say "$1.00." Thus I have counted out enough with the $1.25 to make $2.00. Then to this I count on $3 more, $1 at a time, saying, as I lay the $1 bills out, "$3," "$4," "$5."

The best method of teaching this subject is to take cardboard, and cut out pieces and mark them 1¢, 2¢, 3¢, 5¢, 10¢,

25¢, 50¢; and take slips of paper marked $1, $2, $5, etc., for bills, and ask a pupil to make change for $3.15 out of a $10 bill, etc., and let him count it out by the cards and slips, as if they were money.

SUBTRACTION.

NOTE. — The pupil should be drilled in giving such explanations of the exercises on this page as that given in the text-book, Ex. 5, p. 43, until he is completely master of the argument.

Let it be borne in mind that *three things* are important; viz., —

1. That the pupil secure a clear comprehension of the *rationale* of each process.

2. That he be able to give the argument in clear and connected language.

3. That he acquire *facility* and *accuracy* in computation.

Different kinds of drill-exercises are required to secure these several ends. See Preface to ARITHMETICAL EXERCISES.

Pages 47-51.

These exercises are designed to teach the *uses* of subtraction, not the *process*. Hence, in explaining, the pupil should tell *why* he subtracts, not *how*.

The same analysis or explanation should be given when the pupil explains from the board, as when he solves a problem mentally. Thus, —

Example 7, Page 48.

Having put the work on the blackboard, as in the margin, the pupil, standing by it (if practicable), and pointing to the numbers as he speaks of them, says, "If a person had 735 miles to travel, and has travelled 93 miles of the journey, he has yet to travel the remainder after 93 miles are subtracted from 735. 735 — 93 = 642. Hence he has yet to travel 642 miles."

BLACKBOARD.
(*Pupil's Name.*)

735
93
642

Be sure that the argument is full and clear; yet avoid stereotyped forms.

Encourage the pupil to solve all the examples he can without the use of the board.

Example 33, Page 50.

EXPLANATION. — "In all I borrowed \$175 + \$340 + \$520, or \$1035. Having paid \$685, there remains unpaid the remainder after \$685 is subtracted from \$1035, or \$350."

BLACKBOARD. (*Pupil's Name.*)

\$175
340
520
\$1035
685
\$350

Example 36, Page 54.

EXPLANATION. — "If I subtract what the man spends during the year from what he receives, the remainder will be what he invests. Now he spent \$2500 + \$370, or \$2870. Hence he had remaining to invest \$3700 — \$2870, or \$830."

BLACKBOARD. (*Pupil's Name.*)

\$2500
370
\$2870
3700
\$830

Observe that the pupil should not feel obliged to write the subtrahend *under* the minuend. He should be equally ready to reverse the order when it is more convenient. Call his attention to the closing remark in the first paragraph of "REASONS," p. 44.

Example 48, Page 51.

"The less of two numbers + the difference between them is the greater. Hence 375 + 117, or 492, is the greater."

MULTIPLICATION.

NOTE. — See remarks under Subtraction on pp. 5, 6.

Pages 59, 60.

These examples are designed to give the pupil practice in explaining (demonstrating) the *process* of multiplication.

To the examples given in the text-book we will add one here, showing the spirit and form of such demonstration.

Example 6.

FULL MODEL OF EXPLANATION TO BE GIVEN BY THE PUPIL AT THE BOARD IN RECITATION. — "I am to multiply 543 by 254. Now 254 times 543 is the same as 4 times 543, + 50 times 543, + 200 times 543. For convenience of having the numbers where I can see them readily, I write the multiplier under the multiplicand. I now take 4 times 543, and find it to be 2172. I get 4 times 543 by taking 4 times each of the parts of 543, viz., 4 times 3, 4 times 4 tens, 4 times 5 hundreds, and adding the results together. I multiply the parts in this order, that is, beginning at units, for convenience in adding the products. Thus, when I multiply 3 units by 4, I find that it makes 12; i.e., 1 ten and 2 units. Hence I can write the 2 units in its proper place, and add the 1 ten to the product of the 4 tens by 4, making 17 tens, etc. Having thus obtained 4 times 543, I proceed to take 50 times 543. To do this, I consider that 50 is a composite number, and that it can be resolved into 5 and 10. Then I proceed to multiply successively by 5 and 10, according to Principle (pupil repeat 66 of text-book). 5 times 543 is 2715, found as before by multiplying the parts of the multiplicand by 5 and adding together the products. Having 5 times 543 (2715), I take 10 times this product by removing each order one place to the left, which is equivalent to annexing a 0. Hence I have 10 times 5 times 543, which is 27150. In like manner I get 200 times 543 by taking 100 times 2 times 543. Two times 543 is 1086, and 100 times 1086 is obtained by writing the number two orders to the left, as though two 0's were annexed. Finally, I obtain 200, 50, and 4 times 543, or 254 times 543, by adding these partial products."

BLACKBOARD. (*Pupil's Name.*)

543
254
2172
2715
1086
137922

NOTE. — Possibly with very young pupils the points in such an argument will have to be brought out by questions from the teacher; but no pupil should be allowed to pass the subject without perceiving the argument, and no pupil of much maturity without being able to give it without being prompted.

Observe that in passing to advanced topics we are not necessarily to cumber our demonstrations with a repetition of all elementary principles. *But all such principles should be repeated till they are entirely familiar, and till they can be given readily.*

For practice in explaining (or demonstrating) it will be well to have frequent exercises in writing out demonstrations. Assign as a lesson a single example to be solved, and the demonstration to be written out by each member of the class. Again: assign two or three examples to be solved and the work brought into class on the slates, the class-exercise to consist wholly in drill in demonstrating.

Pages 61-68.

Bear in mind that it should be the purpose of the explanation of the solution of such exercises to give the reasons for multiplying, not the reasons for the manner of the process.

Encourage pupils to solve mentally as many of the problems as they can; but bear in mind that *the explanation should be exactly the same whether the work be performed with or without the use of the board.*

Sufficient models of explanations are given in the text-book. See Ex. 1 and 3 for the more simple cases. We give a few as specimens of the more complex.

Example 51, Page 67.

WORK AS IT WILL STAND ON BLACKBOARD.

John Smith.

$325		$58	
45		75	
1625		290	
1300		406	
$14625	Cost of purchase.	$4350	2d payment.
5225	Total payments.	875	1st payment.
$9400	Unpaid balance.	$5225	Total payments.

EXPLANATION TO BE GIVEN BY PUPIL. — After reading the example, the pupil says, "If I find the entire cost of the farm, and from this subtract all that was paid, the remainder will be what the farmer still owed. Now 325 acres at $1 per acre would cost $325, and at $45 would cost 45 times as much,[1] or 45 times $325, which is $14625, the cost of the purchase. Again: the 1st payment was $875, and the 2d the avails of 75 acres of land at $58 per acre. Now, if 1 acre brought $58, 75 acres brought 75 times as much. 75 times $58 is $4350. Hence the total of the payments is $4350 + $875, or $5225; and there remains unpaid the remainder after subtracting the total of the payments from the entire cost; i.e., $14625 — $5225, or $9400."

Example 54, Page 67.

EXPLANATION. — "If in open air a sheep eats 1912 lb. of turnips, and under cover 1394 lb., the difference between 1912 lb. and 1394 lb. (i.e., 518 lb.) is saved by keeping him under cover. Again: if 518 lb. is saved on 1 sheep, on 345 sheep there will be saved 345 times 518 lb., or 178710 lb."

BLACKBOARD.

Peter White.

1912	
1394	
518	Saved on 1 sheep.
345	
2590	
2072	
1554	
178710	Saved on 345 sheep.

[1] If preferred, the explanation which makes 325 the multiplier can be given. The author prefers making the logic and process agree.

DIVISION.

Page 73.

These examples are designed to teach the principles of what is known as "Short Division." We give a single example as a model for pupil's demonstration.

EXPLANATION. — "I write the divisor on the right of the dividend in accordance with custom, and because it is thus in a position to be seen readily at the same time with the dividend. I divide 5462 by 7, on the principle that we may find how many times one number is contained in another by finding how many times it is contained in each of its parts, and adding these results together. I begin at the left hand, i.e., with the highest order; for in so doing any remainders arising from dividing the higher orders can be readily added in with the next lower, and divided. Thus 7 is contained in 54 7 times, with 5 remainder: hence it is contained in 54 *hundreds* 7 hundred times, with 5 hundreds remainder. (Pupil repeat the Principle, Art. 77.) I therefore write the 7 hundreds in the order of hundreds in the quotient, and add the remainder, 5 hundreds, to the 6 tens, making 56 tens. 7 is contained in 56 8 times, and hence in 56 tens 8 tens times (77). I therefore write the 8 tens in its proper order in the quotient, thus adding it to the part of the quotient already found. Finally, 7 is contained in 2 0 times, with 2 remaining; and I write 0 in units order, and leave the 2 units undivided as a remainder. The quotient is, therefore, 780 with 2 as a remainder."

BLACKBOARD.
James Jones.

7) 5462—2
780

Such analyses as that given on page 72 of the text-book will be found very serviceable in giving a clear insight into the principles of the process; *but the ultimate should be to explain the actual operation just as we perform it*, and not some other form of operation.

Page 80.

The explanation required for examples in "Long Division" is exactly the same as that required in "Short Division." See above.

Pages 84-92.

See foot-note in text-book, p. 84; also solutions of Examples 1 and 2 on that page. We insert here a few sample explanations of somewhat more complex examples.

Example 31, Page 86.

BLACKBOARD.

3268	0	76	0	986	0	34	0[1]
304		43		68		29	
228				306			
228				306			

EXPLANATION. — (The pupil should first read the problem.) "I find here two problems. 1st, If 340 head of cattle cost $8960, what cost 1 head? One head costs 1-340th part as much as 340 head. Hence I divide $9860 by 340, since division is a process of separating a number into any required number of equal parts, to find how many there are in each part (69*a*). Thus I find that 1-340th part of $9860 is $29; and hence 1 head of cattle costs $29. 2d, The second question is: If 760 acres of land cost $32680, what does 1 acre cost? At $1 an acre 760 acres would cost $760: hence the price of 1 acre must have been as many times $1 as $760 is contained times in $32680. $760 is contained 43 times in $3268. Hence the land cost $43 per acre."

We give here both forms of solution in order that the teacher may select. We think it well that the pupil be required to explain in either way. The same example should frequently be explained in both ways.

[1] This form is recommended in preference to the common one.

Example 35, Page 87.

EXPLANATION. — "I first find how many gallons run into the cistern in 1 hour from all the pipes. From 1 pipe 200 gallons run in in 1 hour, and from each of the other two 150 gallons, or 300 gallons from both. Hence 500 run in in 1 hour. If 500 gallons run in in 1 hour, to secure 10000 gallons will require as many hours as 500 is contained times in 10000, or 20 hours."

BLACKBOARD.

200
300
500) 10000
20

Example 57, Page 89.

EXPLANATION. — "If 1 man cut 7 acres in 1 day, 3 men can cut 3 times 7, or 21 acres in 1 day. Hence to cut 250 acres 3 men will require as many days as 21 is contained times in 250, or $11\frac{19}{21}$ days."

OPERATION.

$3 \times 7 = 21$

21) 250 ($11\frac{19}{21}$
21
40
21
19

Example 78, Page 90.

EXPLANATION. — "I first find what the butter and eggs amounted to. 5 doz. eggs at 28¢ per dozen is 5 times 28¢, or 140¢. 28 lb. of butter at 32¢ per lb., is 28 times 32¢, or 896¢.[1] Hence the butter and eggs amounted to 140¢ + 896¢, or 1036¢. Then I find how much the purchases amounted to. 12 yd. of calico at 15¢ per yd. is 180¢, to which adding $7, or 700¢, I have 880¢ as the sum of the purchases. There is therefore a balance of 1036 — 880, or 156¢ coming to the woman. 156¢ will buy as many pounds of sugar at 9¢ per lb. as 9 is contained times in 156, or $17\frac{1}{3}$ lb."

OPERATION.

$5 \times 28 = 140$
$28 \times 32 = 896$ 1036

$12 \times 15 = 180$
700 880

9) 156
$17\frac{1}{3}$

[1] It may be well to show the pupil how to perform such a multiplication thus, without writing the numbers down in the usual way, in fact doing no more writing than appears in the solution: 2 times 8 is 16; 2 times 2 is 4 and 1 is 5, and 3 times 8 is 24 and 5 is 29; 3 times 2 is 6 and 2 is 8.

Example 96, Page 92.

EXPLANATION. — The difference for 1 day between spending 2¢ more than one earns and 3¢ less is 5¢. Hence in any number of days the difference is 5 times as many cents as days. In a year of 365 days there are, exclusive of the 52 Sundays, 313 days, and 5 times 313 is 23475. Hence in 15 years of 313 days each, exclusive of Sundays, the difference between spending 2¢ per day more than one earns, or 3¢ per day less, is 23475¢, or $234.75.

OPERATION.

365
52
313
15
1565
313
4695
5
23475¢ = $234.75

Example 100, Page 92.

The following is a good form for the pupil to use in putting such a solution on the blackboard: —

20 ×	$125 =	$2500
30 ×	100 =	3000
5 ×	400 =	2000
40 ×	150 =	6000
5 ×	85 =	425
Total cost		$13925
Add the gain		500
Sold the 100 for		$14425

Hence the average price of each was $\$\frac{14425}{100} = \$144.25$.

COMMON DIVISORS.

Example 19, Page 96.

The solution of these examples is so simple that little aid can be needed. We suggest that the pupil be trained in the use of such principles as are contained in **96, 97, 98,** and others which the teacher may add, by which to detect the

factor which will divide a number. A good form of solution for the blackboard is, —

5)1100		2)144		5)960	
5)220	(98)	2)72	(96)	2)192	(98)
2)44	"	2)36	"	2)96	(96)
2)22	(96)	2)18	"	2)48	"
11	"	3)9		2)24	"
		3		2)12	"
				2)6	"
				2)3	"

Hence $1100 = 5 \times 5 \times 2 \times 11$.

$144 = 2 \times 3 \times 3 \times 2 \times 2 \times 2.$

$960 = 5 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2.$

Thus we see that 2 is the only prime factor common to all, and therefore is the G. C. D.

COMMON MULTIPLES.

Example 17, Page 98.

We give a single example, as suggesting a form of solution for the blackboard: —

5)45	2)54	3)27	2)18
3)9	3)27	3)9	3)9
3	3)9	3	3
	3		

$45 \times 2 \times 3 = 270$, the L. C. M.

For form of explanation, see text-book.

CANCELLATION.

Page 99.

2. $\frac{\overset{2}{\cancel{16}} \times \overset{12}{\cancel{24}} \times \overset{2}{\cancel{48}}}{\underset{2}{\cancel{32}} \times \underset{3}{\cancel{36}} \times \cancel{9}} = 2$. This is the form of work as it should appear on the blackboard. For the *explanation* to be given by the pupil, see text-book.

6. $\frac{\overset{8}{\cancel{184}} \times \overset{5}{\cancel{145}} \times \overset{4}{\cancel{80}}}{\underset{23}{\cancel{23}} \times \underset{29}{\cancel{29}} \times \underset{3}{\cancel{60}}} = \frac{160}{3} = 53\frac{1}{3}$. Form of work for blackboard.

16. $\frac{\overset{3}{\cancel{21}} \times \overset{2}{\cancel{12}} \times \overset{14}{\cancel{28}} \times \overset{7}{\cancel{35}}}{\underset{2}{\cancel{14}} \times \underset{6}{\cancel{18}} \times \underset{5}{\cancel{20}} \times \underset{5}{\cancel{25}}} = \frac{98}{25} = 3\frac{23}{25}$. The proficient would naturally write $\frac{92}{25} = 3.92$, simply reducing to hundredths by multiplying the terms of the fraction by 4, and pointing off two figures. Students should be encouraged to give such solutions, especially in review.

USE OF SIGNS +, −, ×, ÷, ETC.

Page 101.

3. $(6+8) \times 5 = 14 \times 5 = 70$.
4. $6+8 \times 5 = 6+40 = 48$.
5. $(8-3) \times 2 = 5 \times 2 = 10$.
6. $8-3 \times 2 = 8-6 = 2$.
7. $8+12 \div 4 = 8+3 = 11$.
8. $(8+12) \div 4 = 20 \div 4 = 5$.
9. $(2+1)\ (7-3) = 3 \times 4 = 12$.
10. $3 \times 8 \div 4 \times 3 = 24 \div 4 \times 3 = 6 \times 3 = 18$.

Observe that where the signs $\times$ and $\div$ are used in succession the operations are to be performed in order.

11. $3 \times 8 \div (4 \times 3) = 24 \div 12 = 2.$

12. $14 - \frac{3 \times 4 - 2 \times 3}{2} = 14 - \frac{12 - 6}{2} = 14 - \frac{6}{2} = 14 - 3 = 11.$

13. $\frac{9+3}{2} + \frac{8-2}{2} = \frac{12}{2} + \frac{6}{2} = 6 + 3 = 9.$

14. $4 \times (5 \times 6 \div 3) + 10 = 4 \times (30 \div 3) + 10 = 4 \times 10 + 10 = 40 + 10 = 50.$

15. $(48 \div 2 + 12) \times 3 + (8 \div 2 + 14 \times 2) = (24 + 12) \times 3 + (4 + 28) = 36 \times 3 + 32 = 108 + 32 = 140.$

16. $48 \div (2 + 12) \times (3 + 8) \div 2 + 14 \times 2 = 48 \div 14 \times 11 \div 2 + 28 = \frac{\overset{12}{\cancel{24}}}{\cancel{14} \times \cancel{2}} + 28 = \frac{132}{7} + 28 = 18\frac{6}{7} + 28 = 46\frac{6}{7}.$

APPLICATIONS.

Pages 101, 102.

2. $\frac{\overset{9}{\cancel{63}} \times 9}{\cancel{7}} = \$81.$ It may appear on first thought that the solution of such examples in this manner requires a knowledge of the methods of multiplying and dividing fractions; but such is not the case. For example, in this instance the pupil first discovers that the analysis requires that he divide 63 by 7 and multiply the quotient by 9, and then, *on the principle that the order* of the operations is *immaterial*, he proceeds as above.

4. $\frac{12 \times 11¢}{120} = 11.$

5. $\frac{25 \times \overset{2}{\cancel{40}}¢}{\$0¢} = 2.$
$20¢$

8. The length of the longest stone that will lay the strips is the greatest common measure, i.e., divisor, of each strip of walk. Hence we are to find the G. C. D. of 273, 294, 567, and 651.

3)273	2)294	3)567	7)651
7)91	7)147	3)189	3)93
13	7)21	3)63	31
	3	7)21	
		3	

The common factors being 3 and 7, the G. C. D. is $3 \times 7 = 21$.

9. Find all the common divisors of 132 and 99.

10. This is the least common multiple of 15, 21, 24, since to hire men for 1 month at \$15 each, and thus exhaust a given sum of money, the \$15 must be a factor of that sum, and so of the others. Now,

$$15 = 3 \times 5,\ 21 = 3 \times 7,\ 24 = 2 \times 2 \times 2 \times 3.$$

$\therefore\ 24 \times 5 \times 7 = 840$ being the L. C. M., \$840 is the sum required.

11. The G. C. D. of 225, 297, and 369 is required, as in the 8th. He might also use panels 3 ft. long.

12. The G. C. D. of 125, 175, and 225 is required.

13. Find the L. C. M. of 14, 30, and 90.

14. Find the G. C. D. of 413, 531, and 590, which is 59. Hence they must pay \$59 per head. Therefore A buys $\frac{413}{59} = 7$; B, $\frac{531}{59} = 9$; and C, $\frac{590}{59} = 10$.

15. A requires $\frac{200}{20} = 10$ days to go around; B, $\frac{200}{25} = 8$; and C, $\frac{200}{40} = 5$. Hence the L. C. M. of 10, 8, and 5 is required.

COMMON FRACTIONS.

Examples for Practice.

Pages 118, 119.

29. $4\frac{3}{7}+\frac{6}{7}-(\frac{3}{7}+\frac{6}{7})$. In solving such an example the pupil should be trained to notice that the $\frac{6}{7}$ of the subtrahend $(\frac{3}{7}+\frac{6}{7})$ just destroys the $\frac{6}{7}$ of the minuend $(4\frac{3}{7}+\frac{6}{7})$, and the $\frac{3}{7}$ of the subtrahend destroys the $\frac{3}{7}$ of the minuend. This may be exhibited thus: —

Minuend	$4\frac{3}{7}+\frac{6}{7}$
Subtrahend	$\frac{3}{7}+\frac{6}{7}$
Remainder	4

The subtraction is thought of thus: $\frac{6}{7}$ from $\frac{6}{7}$ leaves 0, $\frac{3}{7}$ from $\frac{3}{7}$ leaves 0. Hence the remainder is 4.

30. Minuend, $24\ -\frac{3}{7}+\frac{5}{9}$

Subtrahend, $9\frac{3}{7}$

$$14\tfrac{4}{7}-\tfrac{3}{7}+\tfrac{5}{9}=14\tfrac{1}{7}+\tfrac{5}{9}=14\tfrac{9}{63}+\tfrac{35}{63}=14\tfrac{44}{63}.$$

Certainly pupils somewhat advanced, as in review, should be taught to use such methods.

31. Taking the $6+9$, or 15 from 113, we have $98+\frac{5}{7}+\frac{3}{14}=98+\frac{12+3}{14}=99\frac{1}{14}$.

32. $\frac{5}{7}+12-11+3-2=\frac{5}{7}+2=2\frac{5}{7}$.

33. $6\frac{7}{8}-\frac{3}{8}=6\frac{1}{2}$. $13-6\frac{1}{2}=6\frac{1}{2}$. Hence we have $\frac{4}{7}+1\frac{1}{3}+6\frac{1}{2}=\frac{4}{7}+7\frac{5}{6}=7+\frac{24+35}{42}=7+\frac{59}{42}=7+1\frac{17}{42}=8\frac{17}{42}$.

This might be written on the blackboard thus: —

$$\tfrac{4}{7}+13-(6\tfrac{7}{8}-\tfrac{3}{8})+\tfrac{4}{3}=13-6\tfrac{1}{2}+1\tfrac{1}{3}+\tfrac{4}{7}=6\tfrac{1}{2}+1\tfrac{1}{3}+\tfrac{4}{7}$$
$$=7\tfrac{5}{6}+\tfrac{4}{7}=7+\frac{35+24}{42}=7\tfrac{59}{42}=8\tfrac{17}{42}.$$

34. $8\frac{3}{7}+11-(3-2)+\frac{5}{7}=8\frac{3}{7}+\frac{5}{7}+11-1=9\frac{1}{7}+10=19\frac{1}{7}$.

35. $(14+\frac{3}{5}+\frac{6}{7}-\frac{3}{4})-4\frac{3}{7}=14\frac{6}{7}-4\frac{3}{7}+\frac{1}{5}-\frac{3}{4}=10\frac{3}{7}+\frac{1}{5}$
$-\frac{3}{4}=10+\frac{36+28-63}{84}=10\frac{1}{84}$.

36. $\frac{5}{8}+(16\frac{3}{7}-\frac{6}{8})-\frac{3}{7}+\frac{1}{8}=16\frac{3}{7}-\frac{3}{7}-\frac{3}{4}+\frac{5}{8}+\frac{2}{8}=15\frac{1}{4}$
$+\frac{5}{8}+\frac{2}{8}=15+\frac{6+15+16}{24}=16\frac{13}{24}$.

37. $\frac{3}{7}+(\frac{5}{8}-\frac{2}{7})+(\frac{3}{4}+\frac{6}{7})=\frac{3}{7}+\frac{6}{7}-\frac{2}{7}+\frac{3}{4}+\frac{5}{8}=\frac{7}{7}+\frac{6}{8}$
$+\frac{5}{8}=2\frac{3}{8}$.

38. $5\frac{2}{6}+\frac{3}{4}-\frac{6}{7}+(\frac{3}{8}+13)=13+5\frac{2}{6}+\frac{3}{4}+\frac{3}{8}-\frac{6}{7}=$
$18\frac{2}{3}+1-\frac{6}{7}+\frac{1}{8}=18+\frac{2}{3}+\frac{1}{7}+\frac{1}{8}=18+\frac{112+72+63}{504}=$
$18\frac{247}{504}$.

39. $6\times5+\frac{3}{4}-4\frac{3}{7}=30-5\frac{1}{7}+\frac{3}{4}=24\frac{6}{7}+\frac{3}{4}=24+$
$\frac{24+21}{28}=25\frac{17}{28}$.

40. $\frac{3}{4}+\frac{5}{6}+\frac{5}{8}-\frac{3}{7}=1-\frac{3}{7}+\frac{3}{8}+\frac{5}{6}=\frac{4}{7}+\frac{3}{8}+\frac{5}{6}=$
$\frac{192+126+280}{336}=\frac{598}{336}=1\frac{262}{336}=1\frac{131}{168}$.

41. $\frac{3}{5}+\frac{6}{7}+\frac{3}{9}+\frac{5}{9}+\frac{3}{7}=1\frac{2}{7}+\frac{8}{9}+\frac{3}{5}=1+\frac{90+280+189}{315}$
$=1\frac{559}{315}=2\frac{244}{315}$.

42. $\frac{4}{7}+\frac{1}{3}+\frac{5}{7}-\frac{6}{7}+\frac{1}{11}=\frac{3}{7}+\frac{1}{3}+\frac{1}{11}=\frac{99+77+21}{231}$
$=\frac{197}{231}$.

43. $\frac{2}{\beta}+6\frac{4}{7}-\frac{2}{\beta}+\frac{6}{7}+\frac{5}{14}=7\frac{3}{7}+\frac{5}{14}=7\frac{11}{14}$.

44. $4+\frac{3}{8}+6\frac{3}{4}-(\frac{3}{4}+\frac{6}{7})=3\frac{1}{7}+\frac{3}{8}+6=9+\frac{8+21}{56}$
$=9\frac{29}{56}$.

45. $\frac{0}{7}+14-(12\frac{2}{3}-\frac{3}{4})=\frac{0}{7}+14-(11\frac{1}{4}+\frac{2}{3})=2\frac{3}{4}+\frac{6}{7}-\frac{2}{3}$
$=2+\frac{63+72-56}{84}=2\frac{79}{84}$.

46. $(\frac{4}{7}+\frac{3}{8})+\frac{3}{7}+(\frac{6}{7}+9)=\frac{4}{7}+\frac{3}{7}+\frac{6}{7}+9\frac{3}{8}=10+\frac{3}{8}$
$+\frac{6}{7}=10+\frac{21+48}{56}=11\frac{13}{56}$.

47. $5\frac{3}{7}+\frac{4}{7}-\frac{3}{8}+(\frac{4}{7}-\frac{3}{7})=6-\frac{3}{8}+(\frac{4}{7}-\frac{3}{7})=4+\frac{5}{8}+\frac{4}{7}$
$+\frac{1}{7}=4+\frac{5}{8}+\frac{5}{7}=4+\frac{75}{56}=5\frac{19}{56}$.

48. $\frac{4}{7}+16-\frac{3}{11}+(\frac{3}{7}-\frac{2}{14})=15\frac{8}{11}+\frac{6}{7}=15+\frac{56+66}{77}$
$=16\frac{45}{77}$.

49. $(\frac{2}{6}+12\frac{1}{2})-\frac{3}{7}+\frac{5}{8}=12+\frac{1}{14}+\frac{3}{8}+\frac{5}{8}=12+$
$\frac{72+224+630}{1008}=12+\frac{926}{1008}=12\frac{463}{504}$.

50. $\frac{5}{8}+\frac{5}{7}-\frac{3}{8}-\frac{4}{7}+\frac{4}{7}=\frac{105-112+96}{168}=\frac{89}{168}$.

51. $(11+\frac{3}{4})+\frac{5}{8}-\frac{3}{8}=11+\frac{3}{8}+\frac{5}{8}=11+\frac{27+40}{72}$
$=11\frac{67}{72}$.

53. $5+\frac{3}{7}+\frac{3}{8}-\frac{2}{3}=4+\frac{1}{3}+\frac{3}{7}+\frac{3}{8}=4+\frac{56+72+63}{168}$
$=4\frac{191}{168}=5\frac{23}{168}$.

54. $(8\div 11)+(3\div 9)=\frac{8}{11}+\frac{1}{3}=\frac{24+11}{33}=1\frac{2}{33}$.

MISCELLANEOUS EXERCISES.

Page 128.

1. $\frac{\frac{7}{8}\text{ of }\frac{4}{7}}{\frac{4}{7}\text{ of }\frac{8}{14}}=\frac{7}{8}\times\frac{\cancel{4}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{4}}\times\frac{7}{4}=\frac{49}{32}=1\frac{17}{32}$.

2. $\frac{\frac{3}{5}\text{ of }2\frac{1}{2}}{1\frac{5}{7}}=\frac{\cancel{3}}{5}\times\frac{11}{5}\times\frac{7}{\cancel{12}_{4}}=\frac{77}{100}$.

3. $\frac{\frac{7}{8}+\frac{4}{7}}{\frac{5}{7}+\frac{1}{6}}=\frac{\frac{147+96}{\cancel{8}\times\cancel{7}\times\cancel{3}}}{\frac{120+28}{\cancel{7}\times\cancel{4}\times\cancel{6}}}=\frac{243}{148}=1\frac{95}{148}$.

See text-book (151) *Rule.*

4. $\frac{\frac{6}{7}-\frac{3}{10}}{\frac{5}{7}-\frac{2}{3}}=\frac{\frac{2}{3}-\frac{3}{10}}{\frac{5}{7}-\frac{2}{3}}=\frac{\frac{140-63}{3\times 10\times 7}}{\frac{150-140}{7\times 3\times 10}}=\frac{77}{10}=7\frac{7}{10}$.

5. $\dfrac{\frac{1}{3}}{\frac{1}{6}} \times \dfrac{9}{\frac{16}{5}} = \dfrac{1}{2} \times \dfrac{9}{4} \times 3 = \dfrac{27}{2} = 13\frac{1}{2}$.

6. $\frac{1}{3} + \dfrac{4\frac{1}{6}}{12\frac{1}{2}} = \frac{7}{9} + \frac{74}{225} = \dfrac{175+74}{225} = 1\frac{24}{225}$.

7. $\dfrac{7\frac{3}{4}}{\frac{1}{5}} - \dfrac{7\frac{1}{2}}{\frac{5}{12}} = \dfrac{155}{4} - \dfrac{90}{5} = 38\frac{3}{4} - 18 = 20\frac{3}{4}$.

8. $\dfrac{4\frac{2}{3} + 6\frac{2}{5}}{12\frac{5}{8} - 3\frac{4}{5}} = \dfrac{\frac{14}{3} + \frac{33}{5} \times \frac{9}{55}}{\frac{101}{8} - \frac{19}{5}} = \dfrac{\frac{14}{3} + \frac{27}{25}}{\frac{505-152}{5\times 8}} = \dfrac{\frac{350+81}{3\times 25}}{\frac{505-152}{5\times 8}} =$

$\dfrac{431}{3 \times 25} \times \dfrac{5 \times 8}{353} = \dfrac{3448}{5295}$.

9. $\dfrac{5\frac{1}{6} \times 2\frac{1}{3}}{4\frac{5}{6} \div \frac{1}{2}} = \frac{21}{6} \times \frac{7}{3} \times \frac{6}{29} \times \frac{1}{2} = \frac{217}{174} = 1\frac{43}{174}$.

10. $\dfrac{3\frac{3}{4}}{7\frac{1}{2}} - \dfrac{\frac{5}{8}}{35} = \frac{1}{2} - \frac{1}{56} = \frac{28}{56} - \frac{1}{56} = \frac{27}{56}$.

$3\frac{3}{4}$ is $\frac{1}{2}$ of $7\frac{1}{2}$. Reject factor 5, and $\dfrac{\frac{5}{8}}{35} = \frac{1}{8} = \frac{1}{56}$.

11. $\left(\dfrac{\frac{2}{3}}{4\frac{1}{2}} \div \dfrac{\frac{2}{5}}{\frac{5}{4}}\right) \div 5\frac{1}{2} = \frac{2}{3} \times \frac{2}{9} \times \frac{5}{5} \times \frac{4}{11} \times 2 = \frac{40}{297}$.

12. $\dfrac{5\frac{2}{5} \div \frac{24}{7}}{19\frac{4}{5} \div 8\frac{4}{13}} = \dfrac{\frac{27}{5} \times \frac{7}{18} \times \frac{39}{4}}{\frac{99}{5} \times \frac{13}{108}} = \dfrac{3}{\frac{27}{5}} \times \dfrac{7}{\frac{18}{2}} \times \dfrac{3}{\frac{39}{4}} \times \dfrac{5}{\frac{99}{11}} \times \dfrac{3}{\frac{27}{108}}$

$= 8\frac{13}{22}$.

13. $\dfrac{12\frac{3}{4} - 7\frac{1}{8}}{\frac{4}{5} \times \frac{2}{5}} = \dfrac{5\frac{5}{8}}{\frac{1}{4}} = \frac{45}{2} = 22\frac{1}{2}$.

PRACTICAL PROBLEMS.

FOR MENTAL SOLUTION.

Pages 128-135.

16. *The practical operation* consists in dividing $\frac{2}{3}$ by $\frac{4}{5}$. Thus, $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$.

The Analysis, which is very important as a means of disciplining the reasoning powers — training to think — is as follows: 1 is $\frac{1}{4}$ of 4; and, as $\frac{4}{5}$ is only $\frac{1}{5}$ as large as 4, 1 is 5 times as large a part of $\frac{4}{5}$ as it is of 4. Hence 1 is 5 times $\frac{1}{4}$, or $\frac{5}{4}$ of $\frac{4}{5}$. Now, as 1 is $\frac{5}{4}$ of $\frac{4}{5}$, and $\frac{2}{3}$ is $\frac{2}{3}$ as large as 1, it is $\frac{2}{3}$ of $\frac{5}{4}$ of $\frac{4}{5}$. $\frac{2}{3}$ of $\frac{5}{4}$ is $\frac{5}{6}$.

REMARK. — That $\frac{2}{3}$ is, or equals, $\frac{2}{3}$ of $\frac{5}{4}$ of $\frac{4}{5}$ is evident, since $\frac{2}{3}$ of $\frac{5}{4}$ of $\frac{4}{5}$ is $\frac{2}{3}$, as $\frac{5}{4}$ of $\frac{4}{5}$ is 1.

17. *Analysis.* 1 is 5 times $\frac{1}{5}$; and, as $\frac{2}{5}$ is 2 times $\frac{1}{5}$, 1 is only $\frac{1}{2}$ as great a part of $\frac{2}{5}$ as it is of $\frac{1}{5}$. $\frac{1}{2}$ of 5 is $\frac{5}{2}$. Hence 1 is $\frac{5}{2}$ times $\frac{2}{5}$. Now, as 1 is $\frac{5}{2}$ of $\frac{2}{5}$, and $\frac{6}{7}$ is $\frac{6}{7}$ of 1, $\frac{6}{7}$ is $\frac{6}{7}$ of $\frac{5}{2}$ of $\frac{2}{5}$. $\frac{6}{7}$ of $\frac{5}{2}$ is $\frac{15}{7}$, or $2\frac{1}{7}$. Hence $\frac{6}{7}$ is $2\frac{1}{7}$ times $\frac{2}{5}$.

REMARK. — As above, it is evident that $\frac{6}{7}$ is, or equals, $\frac{6}{7}$ of $\frac{5}{2}$ of $\frac{2}{5}$, since the $\frac{5}{2}$ of $\frac{2}{5}$ is 1.

23. One week is $\frac{1}{52}$ of a year. Hence 13 weeks is 13 times $\frac{1}{52}$, or $\frac{1}{4}$ of a year.

NOTE. — The teacher should be exacting in requiring that the pupil perform the operations in the most expeditious way. Thus, in this case, he should not be allowed to say, "13 times $\frac{1}{52}$ is $\frac{13}{52}$, which equals $\frac{1}{4}$," but should perform the multiplication by dividing the denominator.

25. Since there are 3 feet in a yard and 12 inches in a foot, there are 12 times as many inches in a yard as there are feet; i.e, 12 times 3, or 36 inches. The question now is, "What part of 36 inches is $4\frac{1}{2}$ inches, etc.?" and hence is the same as above.

REMARK. — The form of analysis which makes the number which

it takes of the lower denomination to make one of the higher the multiplier, is preferred, since in practice in reduction this is practically made the multiplier. Nevertheless, before leaving the subject, it should be clear to the pupil that either number may be made the multiplier in the analysis, as well as in the process. *But let one method be clearly seen before any attempt is made to give another.* Such questions as the following will lead to a comprehension of the subject: "Here is a stick which is 2 feet long; is it more or less than 2 *inches* long?" "How many times as many inches as feet in a given length?" etc.

27. Since 8 oz. $= \frac{1}{2}$ lb., 5 lb. 8 oz. cost $5\frac{1}{2}$ times $20¢ = \$1.10$.

7 lb. cost 7 times $20¢ = \$1.40$; and, since 4 oz. $= \frac{1}{4}$ lb., 4 oz. cost $\frac{1}{4}$ of $20¢ = 5¢$. $\therefore$ 7 lb. 4 oz. cost \$1.45.

Since 12 oz. $= \frac{3}{4}$ lb., 2 lb. 12 oz. cost $2\frac{3}{4}$ times $20¢ = 55¢$.

Let such mental solutions be as brief as is consistent with entire perspicuity: multiplication of words by no means tends to clearness of conception.

29. $\frac{1}{5}$ of $\frac{2}{5} = \frac{1}{5}$. Hence $\frac{2}{5}$ and $\frac{1}{2}$ of $\frac{2}{5} = \frac{3}{5}$. $\frac{3}{5}$ of $12 = \frac{36}{5} = 7\frac{1}{5}$.

The full analysis of $\frac{3}{5}$ of $12 = \frac{36}{5}$ is, $\frac{3}{5}$ of 1 is $\frac{3}{5}$, and $\frac{3}{5}$ of 12 is 12 times as much, or $\frac{36}{5}$.

33. 1 mo. is $\frac{1}{12}$ of a year. $\therefore$ 3 mo. $= \frac{3}{12} = \frac{1}{4}$ yr.

36. As many yards as 2 is contained times in 5; i.e., $2\frac{1}{2}$ yds.

37. $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$. Hence $\frac{3}{4}$ is greater than $\frac{2}{3}$.

38. $\frac{1}{2}$ is $\frac{3}{6}$. Hence $\frac{5}{6}$ is $\frac{2}{6}$, or $\frac{1}{3}$, more than $\frac{3}{6}$. $\frac{1}{3}$ of a year is 4 mo.

48. There are 8 eighths in 1, and in $\frac{1}{4}$ there are $\frac{1}{4}$ of 8 eighths, or 2 eighths.

51. $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$; and $1 - \frac{2}{5} = \frac{3}{5}$. $\therefore$ A owns $\frac{3}{5}$.

52. $\frac{4}{5} - \frac{1}{3}$ of $\frac{4}{5} = \frac{12}{15} - \frac{4}{15} = \frac{8}{15}$; and $1 - \frac{8}{15} = \frac{7}{15}$. $\therefore$ I have $\frac{7}{15}$ left.

53. The remainder is $1 - \frac{2}{3} = \frac{1}{3}$. $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$. Hence I own $\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$.

(Do not make solutions wordy.)

55. After selling $\frac{5}{7}$, he had remaining $\frac{2}{7}$; and, as he then bought back $\frac{1}{3}$ of $\frac{5}{7}$, he had $\frac{2}{7} + \frac{1}{3}$ of $\frac{5}{7} = \frac{6+5}{7 \times 3} = \frac{11}{21}$.

56. $\frac{4}{5}$ of $\frac{5}{8} = \frac{5}{8}$. Hence B bought $\frac{7}{8}$ of $\frac{4}{9} = \frac{7}{18}$; and, as B bought $\frac{5}{8}$, or $\frac{10}{18}$, he bought the most.

60. If 27 was $\frac{3}{10}$, $\frac{1}{10}$ was $\frac{1}{3}$ of $27 = 9$, and $\frac{10}{10}$ was 90. Hence the farm at first was 90 acres; and, after selling 27 acres, Mr. Whipple had $90 - 27 = 63$ acres left.

84. If 80 acres is $\frac{3}{4}$, $\frac{1}{4}$ is $\frac{80}{3}$, and $\frac{4}{4}$ is $\frac{4 \times 80}{3}$. $\frac{2}{3}$ of $\frac{4 \times 80}{3}$ $= \frac{640}{9} = 71\frac{1}{9}$.

85. If $\frac{1}{3}$ of 10 were 3, $\frac{1}{3}$ of 40 would be 12. Now $\frac{1}{3} = \frac{4}{12}$, and $\frac{1}{4} = \frac{3}{12}$. Hence $\frac{1}{4}$ is $\frac{3}{4}$ of $\frac{1}{3}$. $\frac{3}{4}$ of 12 is 9.

86. The number is 9; and $\frac{1}{4}$ of 4 times a number is the number itself.

87. $60 \times 2\frac{3}{4} = 120 + 45 = 165$, \$1.65.

90. John does $\frac{1}{5}$ and James $\frac{1}{4}$ in a day. Hence in 6 da. they together do $\frac{6}{5} + \frac{6}{4}$ (or $\frac{3}{2}$). $\frac{6}{5} + \frac{2}{3} = \frac{28}{15} = 1\frac{13}{15}$.

91. The 1st discharges $\frac{1}{3}$ of the contents of the vat in 1 hr.; the 2d, $\frac{1}{2}$; and the 3d, $\frac{1}{4}$. Hence running together they discharge $\frac{1}{3} + \frac{1}{2} + \frac{1}{4} = \frac{4+6+3}{12} = \frac{13}{12}$ in an hour. Therefore, to discharge $\frac{1}{12}$ requires $\frac{1}{13}$ of an hour, and to discharge $\frac{12}{12}$, $\frac{12}{13}$ of an hour $= \frac{12}{13}$ of 60 min. $= \frac{720}{13} = 55\frac{5}{13}$ min.

92. As $\frac{1}{7}$ of $3\frac{1}{2}$ is $\frac{1}{2}$, I sell at \$4 per yd., and hence sell $2\frac{1}{2}$ yd. for $2\frac{1}{2}$ times $4 = \$10$.

93. $\frac{1}{3}$ of 27 is 3, and $27 + 3 = 30$. Hence I must sell 100 lb. for \$30, or 3000¢, and 1 lb. for $\frac{1}{100}$ of $3000¢ = 30¢$.

95. Divide by 4.

97. Multiply by 100, and then divide by 8.

100. Ans., 25.

101. Ans., $33\frac{1}{3}$, and 8.

PRACTICAL PROBLEMS.

Written Exercises.

Pages 135-142.

Example 31.

If this example is solved on the board, the *form* in the margin will be found serviceable.

$6\frac{3}{5} = \$6\frac{12}{20}$
$5\frac{1}{2} = \$5\frac{10}{20}$
$8\frac{3}{4} = \$8\frac{15}{20}$
$\$20\frac{17}{20} = \$20.57—.$

Example 33.

Suggestions. — 1st, Seek the best form for putting work on the board. 2d, Do not allow such numbers to be reduced to improper fractions. 3d, The fractions $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}$, should be added by noticing that the two $\frac{1}{2}$'s make 1, and the $\frac{3}{4}$ and one of the $\frac{1}{4}$'s make 1. Hence altogether their sum is $2\frac{1}{4}$. This should be seen at a glance. 4th, In practical operations seek the most expeditious methods.

Form for Blackboard.

$33\frac{1}{2}$
$27\frac{3}{4}$
$40\frac{1}{4}$
$35\frac{1}{2}$
$29\frac{1}{4}$
$166\frac{1}{4}$ bush. bought.
$140\frac{1}{4}$ " sold.
26 " left.

$12\frac{3}{4} \times 3 = 38\frac{1}{4}$
$25\frac{1}{2} \times 4 = 102$
Bush. sold, $140\frac{1}{4}$

Example 48.

$19\frac{4}{5}$ $\frac{16}{20}$
$10\frac{1}{4}$ $\frac{5}{20}$
$9\frac{11}{20}$

$45
$10\frac{1}{4}$
450
11.25
$461.25
410.65
$871.90

$43
$9\frac{11}{20}$
387
23.65
$410.65

47.3

The above are the actual figures which it will be convenient to write. Thus the $\frac{16}{20}$ and $\frac{5}{20}$ are simply the $\frac{4}{5}$ and $\frac{1}{4}$ with a common denominator. The 47.3 standing alone is 11 times 43, divided by 10. Then, dividing 47.3 by 2, we get 23.65. *This is not the explanation to be given in class, however.* That would be somewhat as follows:

"Having sold $10\frac{1}{4}$ tons, I had remaining $19\frac{1}{2} - 10\frac{1}{4}$ tons, or $9\frac{11}{20}$. $10\frac{1}{4}$ tons at \$45 per ton amounts to \$461.25; and $9\frac{11}{20}$ tons at \$43 amounts to \$410.65. Hence in all I received \$461.25 + \$410.65, or \$871.90."

36. The most convenient practical method of solving this is to consider that $\$3\frac{1}{2}$ is \$3.50. Then we have $\frac{3}{4}$ of \$15 = \$11.25. $\frac{2}{7}$ of 350 is found equally well by taking $\frac{1}{7}$ of 700, or 2 times $\frac{1}{7}$ of 3.50; i.e., $2 \times .50$. $32\frac{1}{2}$ is multiplied by $15\frac{1}{2}$ by taking $\frac{1}{2}$ of $32\frac{1}{2}$, which is $16\frac{1}{4}$; and then saying 15 times $\frac{1}{2}$ is $7\frac{1}{2}$, writing the $\frac{1}{2}$ and carrying the 7. 15 times 2 is 30, and 7 makes 37. 15 times 3 is 45, and 3 makes 48, etc.

\$3.50	$32\frac{1}{2}$
$5\frac{3}{7}$	$15\frac{1}{2}$
100	$16\frac{1}{4}$
17.50	$487\frac{1}{2}$
18.50	$\$5.03\frac{3}{4}$
$16.28\frac{4}{7}$	11.25
$\$2.21\frac{3}{7}$	$\$16.28\frac{3}{4}$

Seek convenient practical methods.

38. The form of practical solution is in the margin.

3)140	
$46\frac{2}{3}$	$93\frac{1}{3}$
$8\frac{2}{3}$	
$31\frac{1}{9}$	
$373\frac{1}{3}$	
$\$4.04\frac{4}{9}$	
4.90	
.85⅔	

As $\frac{1}{3}$ dried away, I sold $\frac{2}{3}$ of 70 lb. $\frac{2}{3}$ of 70 lb. is $\frac{1}{3}$ of 140 lb. Hence I sold $46\frac{2}{3}$. This, at $8\frac{2}{3}$ cents per lb., is $8\frac{2}{3}$ times $46\frac{2}{3}$ cents. (This multiplication is effected by taking $\frac{1}{3}$ of twice $46\frac{2}{3}$. The $93\frac{1}{3}$ at the right is 2 times $46\frac{2}{3}$, written there for convenience. The multiplication by 8 is effected thus: 8 times $\frac{2}{3}$ is $\frac{16}{3} = 5\frac{1}{3}$, etc.) Do not allow such examples to be solved by reducing the numbers to improper fractions.

39. $5\frac{1}{2} + \frac{3}{6} + \frac{2}{6} + \frac{1}{2} = 7$. Hence the wood in the shed cost me \$7 per cord. Observe in adding the fractions that $5\frac{1}{2} + \frac{1}{2} = 6$, and $\frac{3}{6} + \frac{2}{6} = 1$.

41. In $2\frac{1}{2}$ weeks' working time there are 15 days. Hence in 1 day he cut $35\frac{3}{5} \div 15$. The most expeditious method of dividing $35\frac{3}{5}$ by 15 is to say, "15 is contained in 35 2 times with remainder 5. $5\frac{3}{5} = \frac{28}{5}$ divided by 15 is $\frac{28}{75}$. Hence $35\frac{3}{5} \div 15 = 2\frac{28}{75}$. Finally, as he earns $\frac{1}{2}$ as many dollars as he cuts cords, we have $\frac{1}{2}$ of $2\frac{28}{75}$, or $1\frac{28}{150}$ dollars = $\$1.18\frac{8}{9}$."

42. The most convenient practical form is the following:—

$$\frac{4\frac{3}{8}}{3\frac{1}{2}} \times 40 = \frac{\overset{5}{\cancel{35}}}{\underset{4}{\cancel{28}}} \times \frac{\overset{10}{\cancel{40}}}{1} = 50$$

$$\frac{26\frac{2}{3}}{20} \times 36\tfrac{1}{2} = \frac{\overset{2}{\cancel{80}} \times 73}{\underset{3}{\cancel{60}} \times \cancel{2}} = 48\tfrac{2}{3}$$

$1\frac{1}{3}$, or $\$1.33\frac{1}{3}$.

EXPLANATION. — Since $3\frac{1}{2}$ yd. cost $\$4\frac{3}{8}$, 1 yd. cost $\frac{4\frac{3}{8}}{3\frac{1}{2}}$, and 40 yd. cost $\frac{4\frac{3}{8}}{3\frac{1}{2}} \times 40$, etc.

43. It takes 5 strips of carpeting, each $5\frac{1}{2}$ yd. long. Hence $5\frac{1}{2} \times 5 = 27\frac{1}{2}$ yd. of carpeting. Hence —

$\$1.10$
$27\frac{1}{2}$
55
2970
$\$30.25$

REMARK. — Such problems are always solved in practice by finding how many "breadths" are required to cover the room in the way the carpet is designed to run, and not by finding the number of square yards to be covered, etc. In this case the methods are essentially the same; but this is not always so.

44. As the persons travel towards each other, the one at the rate of 25 mi. per da. and the other 32 mi., they lessen the distance between them $25 + 32$, or 57 mi. per da. Hence, to lessen that distance 130 mi. (i.e., to come together), they must travel $\frac{130}{57}$, or $2\frac{16}{57}$ da.

45. As they travel in the same direction (one at $4\frac{1}{2}$ mi. per hr., and the other at $3\frac{2}{3}$ mi.), the distance between them will change $4\frac{1}{2} - 3\frac{2}{3}$, or $\frac{5}{6}$ mi. per hr.; and in 5 hr. it will change 5 times $\frac{5}{6}$, or $6\frac{1}{4}$ mi. Whether the distance between them increases or lessens will depend upon whether the foremost man travels the faster or the slower. If the foremost

man travels at $4\frac{1}{2}$ mi. per hr., the distance at the end of 5 hr. will be $7\frac{1}{3} + 4\frac{1}{6}$, or $11\frac{1}{2}$ mi.; but, if the man foremost at the start travels the $3\frac{2}{3}$ mi. per hr., the distance between them at the end of 5 hr. will be $7\frac{1}{3} - 4\frac{1}{6}$, or $3\frac{1}{6}$ mi.

46. Travelling *towards* each other (the one at $2\frac{2}{5}$ and the other at $3\frac{2}{3}$ mi. per hr.), they diminish the distance between them $2\frac{2}{5} + 3\frac{2}{3}$, or $6\frac{4}{15}$ mi. per hr. Hence in 6 hr. this would amount to 6 times $6\frac{4}{15}$, or $37\frac{3}{5}$ mi. But, as they were but 15 mi. apart at starting, 15 mi. of this $37\frac{3}{5}$ would be required to bring them together, when the remainder, $37\frac{3}{5} - 15 = 22\frac{3}{5}$, would be expended in separating them.

47. 1 bush. cost $\frac{\$142}{35\frac{1}{2}}$. Selling at a profit of $\$1\frac{1}{4}$ per bush., he sold at $\frac{\$142}{35\frac{1}{2}} + \$1\frac{1}{4}$ per bush. But he sold $\frac{35\frac{1}{2}}{4}$ bush. Hence he received for what he sold

$$\left(\frac{142}{35\frac{1}{2}} + \frac{5}{4}\right)\frac{35\frac{1}{2}}{4} = \frac{142}{4} + \frac{177\frac{1}{2}}{16} = \frac{1136 + 355}{32} = \frac{1491}{32} = 46\tfrac{19}{32}$$

dollars.

Such a method of performing the operations may not be sufficiently elementary for the youngest pupils, but is of a type which advanced pupils should seek to imitate.

The younger pupils may find it necessary, in order to comprehend the work, to unite the two fractions $\frac{142}{35\frac{1}{2}} + 1\frac{1}{4}$, before multiplying by $\frac{35\frac{1}{2}}{4}$, or $\frac{71}{8}$. But it is questionable whether it would not be better to endeavor to have all apprehend the more expeditious method.

48. The most expeditious form of performing the operation is this: —

$$\frac{617}{11\frac{1}{4}}\left(\frac{\overset{3}{\cancel{6}} \times 7}{\underset{4}{\cancel{8}}} + \frac{4 \times 3}{4} \times \frac{8 \times 2}{3}\right) = \frac{495}{90}\left(\frac{63 + 36 + 64}{12}\right) =$$

$$\tfrac{11}{2} \times \tfrac{163}{12} = \tfrac{1723}{24} = 74.70\tfrac{5}{6}.$$

$\frac{61\frac{7}{8}}{11\frac{1}{4}}$ is the cost of 1 cord. $\frac{6\times 7}{8}+\frac{4\times 3}{4}+\frac{8\times 2}{3}$ is the number of cords.

49. The cloth cost $\frac{260}{40}$, or $\frac{13}{2}$ dollars per yd. $\frac{3}{5}$ of 40, or 24 yd., were sold at $\frac{13}{2}+\frac{3}{8}$, or $\frac{55}{8}$ dollars per yd., and hence amounted to $\frac{55}{8}\times 24=55\times 3=\$165$. The remainder, $\frac{2}{5}$ of 40, or 16 yd., was sold at $\frac{13}{2}-\frac{1}{8}$, or $\frac{51}{8}$ dollars per yd., and hence brought $\frac{51}{8}\times 16=\$102$. The entire receipts were, therefore, $\$165+\$102=\$267$. Hence there was a gain of $\$267-\$260$, or $\$7$.

The above would be the ordinary form of solution. The following is more elegant: —

$$[(\tfrac{260}{40}+\tfrac{3}{8})\ \tfrac{3}{5}\text{ of }40+(\tfrac{260}{40}-\tfrac{1}{8})\ \tfrac{2}{5}\text{ of }40]-260=(\tfrac{55}{8}\times 24+\tfrac{51}{8}\times 16)-260=165+102-260=7.$$

Such forms of operation are secured by indicating all the operations in the first place, and then observing how these indicated operations can be best performed.

50. $11\times 4\frac{1}{4}-8\frac{3}{4}\times 4=46\frac{3}{4}-35=11\frac{3}{4}$.

51. $\$7\frac{1}{2}\quad\frac{4}{8}$

$\$\frac{6\frac{7}{8}}{\frac{5}{8}}\times 49=\$\frac{245}{8}=\$30\frac{5}{8}$.

$\$7\frac{1}{8}$

$\$\frac{6\frac{7}{8}}{\frac{1}{4}}\times 51=\$\frac{51}{4}=\$12\frac{3}{8}$

$\$43\frac{3}{8}=\$43.37\frac{1}{2}$.

52. $\frac{1}{3}$ the longer $=\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$ of the shorter. Then $\frac{3}{8}$ the longer $=3\times\frac{3}{8}=\frac{9}{8}$ the shorter. The shorter being $\frac{8}{8}$ and the longer $\frac{9}{8}$, of the shorter, $\frac{8}{17}$ of $136=64$ is the shorter, and $\frac{9}{17}$ of $136=72$ is the longer.

53. The yield of the 2d, being called $\frac{5}{8}$ that of the first, is $\frac{3}{5}$. Therefore $\frac{3}{8}$ of $344=129$ is the yield of the 1st, and $\frac{5}{8}$ of $344=215$ is the yield of the 2d.

54. A owns $\frac{3}{17}$; B, $\frac{15}{29}$ of $\frac{14}{17}=\frac{210}{493}$; and C, $\frac{3}{10}$ of $(\frac{3}{17}+\frac{210}{493})=\frac{891}{4930}$. $\frac{3}{17}=\frac{870}{4930}$. Hence A, B, and C together own

$\frac{870}{4930}+\frac{2100}{4930}+\frac{891}{4930}=\frac{3861}{4930}$, and D owns $\frac{1068}{4930}$. $\frac{870}{4930}$ of \$493000 $=$ \$87000, A's part; $\frac{2100}{4930}$ of \$493000 $=$ \$210000, B's part; $\frac{891}{4930}$ of \$493000 $=$ \$89100, C's part; and $\frac{1068}{4930}$ of \$493000 $=$ \$106900, D's part.

55.
$$\begin{array}{r} 25\frac{7}{16}-5\frac{3}{16}=20\frac{1}{4} \\ 29\ \frac{3}{4}\ -5\ \frac{3}{4}\ =24 \\ 27\ \frac{1}{8}\ -5\ \frac{7}{8}\ =21\frac{1}{4} \\ \hline 65\frac{1}{2} \\ 23\frac{3}{4} \\ \hline 117\frac{7}{8} \\ 32\frac{1}{2} \\ 16\frac{1}{4} \\ 195 \\ 130 \\ \hline \$15.55\frac{5}{8} \end{array}$$

61. Operation. $1\frac{3}{7}\times\frac{7}{2}\times 5\frac{2}{3}=\frac{\overset{5}{\cancel{10}}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{2}}\times\frac{17}{3}=\frac{85}{3}$ $=28\frac{1}{3}$.

62. $\frac{2}{7}$ of 6 yd. is $\frac{12}{7}$ yd. If $\frac{12}{7}$ yd. cost \$2$\frac{3}{7}$, or \$$\frac{17}{7}$, 1 yd. cost \$$\frac{17}{7}\div\frac{12}{7}=$ \$$\frac{17}{12}$. (Perform the division by dividing the denominator of dividend by 7 and multiplying the quotient by 12.) Finally, if 1 yd. cost \$$\frac{17}{12}$, $\frac{3}{5}$ of 7 yd., or $\frac{21}{5}$ yd., cost $\frac{\overset{7}{\cancel{21}}}{\underset{4}{\cancel{5}}}$ of \$$\frac{\overset{4}{\cancel{17}}}{\cancel{12}}=$ \$$\frac{119}{20}=$ \$5.95.

67. $\underbrace{\left(\underbrace{\frac{1}{2}\text{ of }\frac{3}{4}\text{ of }\frac{2}{3}\text{ of }4}_{\text{Whole cost.}}\div\underbrace{\frac{6}{8}\text{ of }\frac{4}{6}\text{ of }\frac{8}{9}\text{ of }\frac{9}{10}\text{ of }3\frac{1}{3}}_{\text{Quantity.}}\right)}_{\text{Cost of unity.}}\times\underbrace{\frac{7}{12}\text{ of }\frac{1}{2}\text{ of }4\frac{4}{5}}_{\text{Quantity.}}=\frac{1}{\cancel{2}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{2}}{\cancel{3}}\times\cancel{4}\times\frac{\cancel{8}}{\cancel{6}}\times\frac{\cancel{6}}{\cancel{4}}\times\frac{\cancel{9}}{\cancel{8}}\times\frac{\cancel{10}}{\cancel{9}}\times\frac{3}{\cancel{10}}$
$\times\frac{7}{\cancel{12}}\times\frac{1}{\cancel{2}}\times\frac{\cancel{24}}{5}=\frac{21}{20}=1\frac{1}{20}$.

70. Time from Chicago to Detroit, $3+6\frac{3}{4}=9\frac{3}{4}$ hr. Spent at stops $2\times 20+12\times 5$, or 100 min. $=1\frac{2}{3}$ hr.

$\therefore$ the running time was $9\frac{3}{4} - 1\frac{2}{3} = 8\frac{1}{12}$; and average rate was $284 \div 8\frac{1}{12} = 284 \times \frac{12}{97} = 35\frac{13}{97}$, or nearly $35\frac{1}{7}$.

72. The form of solution is as follows:

$$\frac{7\frac{1}{2}}{20 \times 5} \times 30 \times 12 = \frac{\overset{3}{\cancel{15}} \times \overset{3}{\cancel{30}} \times \overset{3}{\cancel{12}}}{\cancel{2} \times \cancel{20} \times \cancel{5}} = 27.$$

EXPLANATION. 20 men in 5 months require 5×20 months' supply. If $7\frac{1}{2}$ bbl. are 100 mo. supply, 1 mo. supply is $\frac{7\frac{1}{2}}{20 \times 5}$. Again, 30 men in 1 yr. require 30×12 mo. supply; but, as $\frac{7\frac{1}{2}}{20 \times 5}$ bbl. is 1 mo. supply, 30×12 mo. supply is $\frac{7\frac{1}{2}}{20 \times 5} \times 30 \times 12$.

Similarly, 17 men 6 mo. require

$$\frac{7\frac{1}{2}}{20 \times 5} \times 17 \times 6 = \frac{\overset{3}{\cancel{15}} \times 17 \times \overset{3}{\cancel{6}}}{\cancel{2} \times 20 \times \cancel{5}} = \frac{153}{20} = 7\tfrac{13}{20}.$$

Finally, if $\frac{7\frac{1}{2}}{20 \times 5}$ bbl. is the monthly allowance, the daily allowance in pounds is

$$\frac{7\frac{1}{2} \times 196}{20 \times 5 \times 30} = \frac{\cancel{15} \times \overset{49}{\cancel{196}}}{\cancel{2} \times 20 \times 5 \times \overset{2}{\cancel{30}}} = .49, \text{ or nearly } \tfrac{1}{2} \text{ bbl.}$$

73.

$$\frac{7 \times 8\cancel{0}}{5\cancel{0}} = \frac{56}{5} = 11\tfrac{1}{5}.$$

$$\frac{10 \times \overset{2}{\cancel{100}}}{\cancel{50}} = 20.$$

$$\frac{13 \times \overset{4}{\cancel{200}}}{\cancel{50}} = 52.$$

$$\frac{\cancel{5} \times \overset{4}{\cancel{8}}}{\cancel{10}} = 4.$$

$$\frac{11 \times 176}{50} = \frac{1936}{50} = 387\tfrac{1}{5}.$$

74. $250\frac{1}{4}$
13
$753\frac{1}{4}$
250
$\$3253\frac{1}{4}$ $250\frac{1}{4}$ A. at \$13.
\$1890 189 A. at \$10.
$\$5143\frac{1}{4}$ Total cost.

\$4263
2331
1901.90
\$8495.90 Received.
5143.25 Paid.
\$3352.65 Gain.

$\frac{2}{3}$ of 189 = 126
$18\frac{1}{2}$
63
1008
126
Sold at $18\frac{1}{2}$, \$2331

$\frac{2}{5}$ of $250\frac{1}{4}$ = 100.1
19
Sold at \$19, \$1901.90

$\frac{1}{3}$ of 189 = 63
$\frac{3}{5}$ of $250\frac{1}{4}$ = $150\frac{3}{20}$
Sold at \$20, $213\frac{3}{20}$
20
\$4263

75.
$320 \times \$25\frac{1}{2} = \$8160$
$275 \times \$31\frac{1}{4} = \$8593\frac{3}{4}$
Amt. paid, $\$16753\frac{3}{4}$

$\frac{3}{7}$ of 595 = 255, $255 \times \$20 = \$5100$
$\frac{4}{7}$ of 595 = 340, $340 \times \$30 \times \$10200$
Received . . . \$15300
Paid $16753\frac{3}{4}$
Lost $\$1453\frac{3}{4}$

76. If $\frac{5}{8}$ of *twice* my age is 30 years, $\frac{5}{8}$ of (once) my age is 15 years, and $\frac{1}{8}$ of my age $\frac{1}{5}$ of 15 years, or 3 years. Whence $\frac{8}{8}$ is 8 times 3 years, or 24 years.

If $\frac{3}{7}$ of *four* times my age is $22\frac{2}{7}$, $\frac{3}{7}$ of my age is $\frac{1}{4}$ of $22\frac{2}{7}$, or $5\frac{4}{7}$; $\frac{1}{7}$ is $\frac{1}{3}$ of $5\frac{4}{7}$, or $1\frac{3}{7}$; and $\frac{7}{7}$ is 7 times $1\frac{3}{7}$, or 13 years.

If $\frac{2}{5}$ of *one-half* my age is 8 yr., $\frac{2}{5}$ of (the whole of) my age is 16 yr.; $\frac{1}{5}$ is $\frac{1}{2}$ of 16 yr., or 8 yr.; and $\frac{5}{5}$ is 5 times 8 yr., or 40 yr.

If 30 yr. is 3 times $\frac{2}{3}$ of my age, $\frac{2}{3}$ of my age is 10 yr.; $\frac{1}{3}$ is $\frac{1}{2}$ of 10, or 5 yr.; and $\frac{3}{3}$ is 3 times 5, or 15 yr.

REMARK. — Such examples as the above can be analyzed in a variety of ways; and the largest liberty should be allowed, consistent with clearness of thought and expression. Thus, $\frac{6}{7}$ of 2 times my age is $\frac{6}{7}$ of my age. Hence 30 yr. is $\frac{6}{7} \cdot \frac{1}{6}$ of 30, or 6, is $\frac{1}{7}$, etc. So, as $\frac{3}{7}$ of 4 is $\frac{1}{7}^2$, $22\frac{3}{7}$, or $\frac{156}{7}$, is $\frac{1}{7}^2$ of my age, etc. Again, as $\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{1}{3}$, 8 is $\frac{1}{3}$ of my age. So, also, as 3 times $\frac{2}{3}$ is 2, 30 is twice my age.

Finally, each may be solved as the last preceding this remark was. Thus, "As 30 is $\frac{6}{7}$, $\frac{1}{7}$ is $\frac{1}{6}$ of 30, or 6; and $\frac{8}{7}$ is 8 times 6, or 48 years. Hence 48 years is twice my age," etc.

77. The most expeditious *form of practical solution* is

$$48 \div (\tfrac{5}{12} \text{ of } \tfrac{3}{4}) = \frac{48 \times \overset{4}{\cancel{12}} \times 4}{5 \times \cancel{3}} = \frac{768}{5} = 153\tfrac{3}{5}.$$

THE ANALYSIS is, $\frac{5}{12}$ of $\frac{3}{4}$ is $\frac{5}{16}$. Now, if 48 is $\frac{5}{16}$, $\frac{1}{16}$ is $\frac{1}{5}$ of 48, or $\frac{48}{5}$, and $\frac{16}{16}$ is 16 times $\frac{48}{5}$, or $\frac{768}{5} = 153\frac{3}{5}$.

78. As either is less than unity, the product is less than either.

79. As either is greater than unity, the product is greater than either.

80, 81. Any fraction is brought nearer unity by adding the same number to each term.

82. As the special train ran 40 mi. per hr., while the regular one ran but 32, the special gains $40 - 32$, or 8 mi. per hr. But the regular train had 2 hr. the start, and, running 32 mi. per hr., was 2×32, or 64 mi. ahead. Then, as the special gained 8 mi. per hr., it would be $\frac{64}{8}$, or 8 hr. in overtaking the regular train. Finally, as the special started at 3 P.M., and took 8 hr. to overtake the other, it was $3 + 8$, or 11 P.M. when they came together. And as the special train ran 40 mi. per hr., in 8 hr. it would run 8×40, or 320 mi. Hence it overtook the regular train 320 mi. from New York.

83. $\frac{7}{10}$ of $15¢ = 10\frac{1}{2}¢$

$$\frac{2 \times 12}{7 + 10} = \frac{24}{17} = 1\tfrac{7}{17}.$$

84. 1 egg weighs $\frac{21\frac{1}{2}}{9}$ oz. $= \frac{21\frac{1}{2}}{9 \times 16}$ lb.; 12 weigh $\frac{21\frac{1}{2} \times 12}{9 \times 16}$.

As this number of pounds sell for $12\frac{1}{2}¢$, 1 lb. sells for $\frac{12\frac{1}{2}}{2\frac{1}{2}1 \times 12} = \frac{25}{43 \times 3} = \frac{300}{43} = 6\frac{42}{43}$.

85. If a dozen weighs $1\frac{1}{2} = \frac{3}{2}$ lb., 1 lb. is $\frac{2}{3}$ of a dozen. Hence 1 lb. should sell for $\frac{2}{3}$ as much as 1 doz.

86. $[1750 \div (\frac{3}{11} \times \frac{1}{9} \times \frac{7}{8})] \times \frac{1}{11} \times \frac{1}{12} =$

$$\frac{\overset{250}{\cancel{1750}} \times 11 \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}} \times 7 \times 11 \times \underset{4}{\cancel{12}}} = 500.$$

He sold $\frac{3}{11} \times \frac{1}{9} \times \frac{7}{7}$ of the vessel for \$1750. Hence he would have sold the whole for $1750 \div (\frac{3}{11} \times \frac{1}{9} \times \frac{7}{8})$ dollars, and $\frac{1}{11}$ of $\frac{1}{12}$ of it for $[1750 \div (\frac{3}{11} \times \frac{1}{9} \times \frac{7}{8})] \times \frac{1}{11}$ of $\frac{1}{12}$.

87. Operation. $\frac{1}{\underset{2}{\cancel{4}}}$ of $\frac{1}{4} \times \overset{5}{\cancel{60}} \times \overset{10}{\cancel{60}} = 50$.

Demonstration. — If in 18 sec. it runs $\frac{1}{4}$ mi., in 1 sec. it runs $\frac{1}{18}$ of $\frac{1}{4}$ mi., in 60 sec., or 1 min., $\frac{1}{18}$ of $\frac{1}{4} \times 60$, and in 1 hr., or 60 min., $\frac{1}{\underset{6}{\cancel{18}}}$ of $\frac{1}{4} \times \overset{5}{\cancel{60}} \times \overset{10}{\cancel{60}} = 50$ mi.

88. We are to ascertain which is the greater of the two fractions, $\frac{21}{23}$ or $\frac{23}{25}$. Reducing to forms having common denominators, we have $\frac{21}{23} = \frac{21 \times 25}{23 \times 25} = \frac{525}{575}$, and $\frac{23}{25} = \frac{23 \times 23}{23 \times 25} = \frac{529}{575}$. Hence $\frac{23}{25}$ is $\frac{529}{575} - \frac{525}{575} = \frac{4}{575}$ greater than $\frac{21}{23}$. Since the second sold $\frac{4}{575}$ of a section more than the first, he sold $\frac{4}{\underset{115}{\cancel{575}}}$ of $\overset{128}{\cancel{640}} = \frac{512}{115} = 4\frac{52}{115}$ acres more.

89. Operation. $\frac{\frac{3}{4} \times \overset{3}{\cancel{12}}}{10\frac{3}{7}} = \frac{3 \times 7 \times 3}{\underset{8}{\cancel{72}}} = \frac{7}{8}$.

Demonstration. — If \$$10\frac{3}{7}$ weighs $\frac{3}{4}$ lb., \$1 weighs $\frac{\frac{3}{4}}{10\frac{3}{7}}$ lb., or $\frac{\frac{3}{4} \times \overset{3}{\cancel{12}}}{10\frac{3}{7}} = \frac{3 \times 7 \times 3}{\underset{8}{\cancel{72}}} = \frac{7}{8}$ oz., as 12 oz. = 1 lb.

98. $\frac{38-5}{2} = 16\frac{1}{2}$, No. days A works.

$16\frac{1}{2} + 5 = 21\frac{1}{2}$ " " B "

$\$1\frac{1}{2} \times 16\frac{1}{2} = \$\frac{3}{2} \times \frac{33}{2} = \$\frac{99}{4} = \$24.75.$

$\$1\frac{1}{2} \times 21\frac{1}{2} = \$\frac{3}{2} \times \frac{43}{2} = \$\frac{129}{4} = \$32.25.$

DECIMAL FRACTIONS.

PRACTICAL PROBLEMS.

Pages 155, 156.

Example 2.

EXPLANATION. — He laid up $20.5 + 17.02 + 31.25 + 27$, or 95.77 rods. The wind blew down and displaced $52.37 + 10.5$, or 62.87 rods. Hence there remained in position $95.77 - 62.87$, or 32.9 rods.

OPERATION.

52.37	20.5
10.5	17.02
62.87	31.25
	27.
	95.77
	62.87
	32.9

4.

$6\frac{1}{4} = 6.25$

$12\frac{1}{2} = 12.5$

$5\frac{3}{8} = 5.375$

$6\frac{5}{8} = 6.625$

$\frac{3}{5} = .6$

$\frac{3}{4} = .75$

Sum, 32.1

5.

$\frac{5}{6} = .83\frac{1}{3}$

.25

10.00

$\frac{2}{5} = .4$

$13\frac{1}{2} = 13.5$

$24.98\frac{1}{3}$

12.86

$12.12\frac{1}{3}$, Diff.

$1\frac{1}{2} = 1.5$

$\frac{4}{5} = .8$

8.0

2.56

12.86

6.

Dis. sailed by steamer.	*By sail-vessel.*
305.24	105
206.5	$204\frac{3}{4} = 204.75$
182.402	$98\frac{1}{2} = 98.5$
343.25	$110\frac{5}{8} = 110.625$
287.13	$212\frac{5}{6} = 212.833\frac{1}{3}$
313.008	$113\frac{2}{5} = 113.4$
1637.53	$845.108\frac{1}{3}$
$845.108\frac{1}{3}$	
$792.421\frac{2}{3}$	

8. *Put into bin in all.*

$87\frac{1}{2} = 87.5$
$33\frac{1}{2} = 33.5$
41.75
162.75
141.50
21.25 $= 25\frac{1}{4}$.

Sold.

$6\frac{3}{4} = 6.75$
$\frac{1}{2} = .5$
17.00
$25\frac{1}{2} = 25.5$
42.25
18.5
31.
141.50

9.
$156.75
428.625
243.125
89.50
$918.
1000.
$82.

10.
.5
1.75
8.00
5.25
21.125
36.625
41.75
5.125 $= 5\frac{1}{8}$

11.
$347.50
1000.00
75.75
15.50
$1438.75
7528
$6089.25

PRACTICAL PROBLEMS.

Pages 161, 162.

2.
$6\frac{3}{4} = \$6.75$
$14\frac{1}{2} = 14.5$
3375
2700
675
$97.875

3.
$17\frac{3}{8} = 17.375$
$11\frac{5}{8}¢ = \$0.11625$
86875
34750
104250
191125
$2.01984375
5.00
$2.98

4. $103\frac{1}{2} \div 5\frac{3}{4} = 103.5 \div 5.75 = 18 = 1\frac{1}{2}$ doz.

5. $\$112.50 \div 12 = \$9.37\frac{1}{2}$.

6. $288 \div 23\frac{3}{5} = 288 \div 23.6 = 12.2+$.

7. $19\frac{1}{2} - \left(3 \times \frac{20}{60} + 10 \times \frac{6}{60}\right) = 17\frac{1}{2}$.

$$\frac{350}{17\frac{1}{2}} = \frac{700}{35} = 20.$$

8. $12 \times 16\frac{1}{2} \times 1\frac{3}{4} = 3 \times \frac{33}{2} \times 7 = \frac{693}{2} = 346.50$.

9. $\frac{69\frac{1}{3}}{130.75} = \frac{208}{392.25} = .53+$.

10. If in 1 hr. a man will dig $\frac{1}{2}$ rd., 3 men will dig $\frac{3}{2}$ rd. Hence, in $3\frac{1}{5}$ da. of $8\frac{1}{2}$ hr. each, 3 men can dig

$3\frac{1}{5} \times 8\frac{1}{2} \times \frac{3}{2} = \frac{\overset{4}{\cancel{17}}}{5} \times \frac{17}{\cancel{2}} \times \frac{3}{\cancel{2}} = \frac{204}{5} = 40\frac{4}{5}$, or $3.2 \times 8.5 \times 1.5 = 40.8$.

GOODS SOLD BY 100, 1000, OR THE TON.

Pages 164–166.

5. The fence is $(16\frac{1}{2} \times 10 + 16\frac{1}{2} \times 20)\ 2 = 990$ ft. Hence $\frac{990}{16}$, or 62 panels, and 2 ft. waste. A panel requires $16 + 10\frac{2}{3} + 26\frac{2}{3} = 53\frac{1}{3}$ sq. ft. of boards. Hence, as $53\frac{1}{3} \times 62 \times 17\frac{1}{4} = 57.04$, it will cost \$57.04.

6. 8750 = 8.75 M. This at \$4.75 per M. costs 8.75×4.75, or \$41.56+.

FOR BLACKBOARD.

8.75
4.75
4375
6 125
35 00
\$41.5625

7.

117.8
$2\frac{1}{2}$
589
235 6
\$294.50

8.

1.875 (2.000
.9375
12.50
468750
11 2500
\$11.718750,
or \$11.72.

9.

$2785\frac{1}{2}$
$3056\frac{3}{4}$
$2907\frac{1}{4}$
3000
$3172\frac{1}{2}$
14922
4750
10172

Solve as above, or follow the rule (182).

10172
$15\frac{1}{2}$
5086
152580
2)157.666
$78.83

14. 58.125 (7.5
52 5
5 62
5 25
375
375

7.75 tons.
2000
15500 lbs.

15. 31000 lb. = 15.5 tons.

116.25 (15.5
108 5
7 75
7 75

$7.50

19. 1200 tons = 1200 × 2000 = 2400000 lb. 2400000 lb. of wheat = $\frac{2400000}{60}$ = 40000 bush. 4000 bush. at $5\frac{1}{2}$¢ per bush. = 40000 × $5\frac{1}{2}$ = 220000¢, or $2200. Hence it is $300 better to pay the freight.

21. 6 crocks at $35\frac{1}{2}$ lb. = $35\frac{1}{2}$ × 6 = 213 lb. To this add the return of the crocks, viz., 4 × 6, or 24 lb., and we have 213 + 24 = 237 lb. on which to pay expressage. This at $1\frac{1}{4}$ per cwt. is 2.37 × $1\frac{1}{4}$ = $2.9625. There were 213 lb. — 24 lb., or 189 lb. of butter. Hence the expressage increases the price $\frac{296.25}{189}$ = 1.56+ cents per lb.

The work as put on the blackboard should be as in the margin.

237 is multiplied by $1\frac{1}{4}$ by adding $\frac{1}{4}$ of 237 to 237. The decimal point is then removed 2 places to the left, because the cost is per cwt. The analysis of this step is, "At $1\frac{1}{4}$ per lb. the expressage would be $296.25. Hence at $1\frac{1}{4}$ per cwt. it is $\frac{1}{100}$ of $296.25, or $2.9625.

$35\frac{1}{2}$
6
213
24 = 6 × 24
237 213
59.25 24
2.9625 (189
1 89 .0156+
1 072
945
1275
1134
141

24. The question is, "If eggs sell at $18\frac{3}{4}$¢ for 12, what

is that for $9\frac{1}{2}$?" One egg sells for $\frac{18\frac{3}{4}}{12}$, and $9\frac{1}{2}$ for $\frac{18\frac{3}{4} \times 9\frac{1}{2}}{12} = \frac{75 \times 19}{2 \times 4 \times 12} = \frac{25 \times 19}{2 \times 4 \times 4} = \frac{475}{32} = 14.84+$, or about 15¢.

25. EXPLANATION. 1576 ems is 1.576 thousand. At 1.576 thousand per page, 296 pp. contain $1.576 \times 296 = 466.496$ thousand. This at 40¢ per thousand is $.40 \times 466.496$, or \$186.60.

OPERATION.

1.576
296
9 456
141 84
315 2
466.496
.40
\$186.59840,
or \$186.60.

26.

\$2537.1 *	\$18.75	210
634.275	1.483	.21
317.137	5625	\$44.10
\$951.412	15000	
27.806	7500	7.342
44.10	1875	$6\frac{1}{2}$
47.723	\$27.80625	3671
\$1071.04		44052
		\$47.723

27. $\frac{.37\frac{1}{2}}{100} = \frac{.375}{100} = .00375$¢ pr. lb. $\frac{.22\frac{1}{2}}{60} = \frac{.225}{60} = .00375$¢ pr. lb.

28. $[1 - (.2 + .35 + .15)]$ of $640 = .3$ of $640 = 192$.

29. $\frac{\overset{50}{\cancel{100}}}{\frac{5}{8} \times 2\frac{1}{2} \times \underset{13}{\cancel{26}}} = \frac{\overset{2}{\cancel{50}}}{\frac{5}{8} \times \frac{5}{2} \times 13} = \frac{32}{13} = 2.461+$.

* Cost at \$100 per M.

26.

Lath.	*Shingles.*	*Scantling.*	*Boards.*
210	7342	1483	25371
21	$6\frac{1}{2}$	$18\frac{3}{4}$	$37\frac{1}{2}$
210	3671	741	12685
420	44052	371	177597
\$44.10	\$47.723	11864	76113
		1483	\$951.412
		\$27.806	27.806
			47.723
			44.10
			\$1071.041

27. $22\frac{1}{2}¢$ per bushel is $\frac{22\frac{1}{2} \times \overset{5}{1}\phi\phi}{\underset{3}{\phi\phi}} = \frac{\overset{15}{45} \times 5}{\underset{2}{\phi}} = \frac{75}{2} = 37\frac{1}{2}$ per 100 lb. Hence the rates are equal.

28. $.2 + .35 + .15 = .7$. Hence the 4th had .3. .3 of $640 = 192$.

29. $\frac{100}{26 \times 2\frac{1}{2} \times .62\frac{1}{2}} = 2.461+$.

DENOMINATE NUMBERS.

Page 171.

17. $\frac{\$0.95\frac{3}{4}}{4} \times 175 = \$41.89\frac{1}{16}$.

18. $125.6 \times \$4.88\frac{1}{2} = \$613.556$. $\frac{1000}{5.15625} = 193.939+$.

Page 179.

5. $\frac{31\frac{1}{2} \times \overset{1}{4} \times 57.75}{\frac{5}{4} \times \underset{2}{\$2} \times 67.2} = \frac{63 \times 57.75}{1\phi \times 2 \times \underset{672}{\$7.2}} = \frac{363825}{1344} = 2.7+$.

7. A liquid quart is $57\frac{3}{4}$ cu. in., and a bushel is 2150.4 cu. in. Hence I received

$$\frac{57\frac{3}{4} \times 32}{2150.4} = \frac{5775}{6720} = \frac{385}{448}.$$

$$\frac{100.44}{\frac{5}{4}} = \frac{401.76}{5} = 80.35+.$$

Or, more directly, as 57.75 cu. in. make a liquid quart, and 67.2 cu. in. make a dry quart, I received only $\frac{57.75}{67.2}$, or $\frac{5775}{6720}$ of what I should, or of 1 bush. He should have given me $\frac{2150.4}{57.75}$ or 37.2+ liquid quarts.

Page 181.

2. 3 oz. gold $= \frac{1}{4}$ lb. $= \frac{1}{4}$ of 5760 gr. $= 1440$ gr.

3 oz. butter $= \frac{3}{16}$ lb. $= \frac{3}{16} \times 7000$ gr. $= 1312\frac{1}{2}$ gr.

Therefore 3 oz. gold is 1440 gr. $- 1312\frac{1}{2}$ gr. $= 127\frac{1}{2}$ gr. the heavier.

3. $\frac{200}{60} = 33\frac{1}{3}$.

12. $\frac{224}{8} = 28$.

14. 1 ton of water $= 27.79 \times 2000 = 55580$ cu. in. But 1 ton of ice contains $\frac{55580}{.93 \times 1728} = 34.46+$ cu. ft.

Pages 184, 185.

2. To-day is Saturday, Sept. 27, 1879. Let us *first* find what day of the week the 4th of this month was; *second*, what day of the week the 4th of July of this year was; and, *third*, what the 4th of July, 1776, was.

First, Passing back 3 wk., 27 — 21, we find that Sept. 6 was Saturday: whence the 4th was *Thursday*.

Second, To pass back to July 4th, we observe that July and August have ended in the interval, each of which has an excess of 3 da. on 28. Hence we are to put back the day of the week 6 da., or to *Friday*, to reach July 4th. July 4th, 1879, was therefore *Friday*.

Third, Now to pass to July 4th, 1776. There have 103 yr. ended in the period. Dividing by 4, we find a remainder 3; and, as 1776 was leap year, we add one, showing the number of leap years to be 26. But 1800 was not leap year: hence, deducting 1, we have the actual number 25. This added to 103; and the sum, 128, divided by 7, gives a remainder 2. Hence the 4th of July was two days earlier in the week in 1776 than in 1879. It was therefore *Wednesday*.

OPERATION.

1879
1776
4)103 — 3
25
1
26
1
25
7)128 — 2
18

4. The 27th day of September, 1879, is Saturday: hence the 25th was *Thursday*. $\frac{2+3+2}{7} = 1$, and 0 remainder. Therefore the 25th of December, 1879, will be *Thursday*.

As the excess of days above exact weeks of the years ending in the period is 3, we are to *advance* the day of the week 3 days from Thursday. Christmas, A.D. 2000, will therefore be Sunday.

2000
1879
4)121 — 1
30
1
29
7)150 — 3
21

5. Between October and January, the 3 mo. ending have an excess of days above 28 of $3+2+3=7$; i.e., an exact week.

6. Beginning with May, the excesses over 28 days are 3, 5, 8, 11, 13, 16, and 18; and, as no one is a multiple of 7, no month in the year after May begins on the same day of the week as May.

Between March and November we have

March, 3
April, 2
May, 3
June, 2
July, 3
Aug., 3
Sept., 2
Oct., 3

$7)21 - 0$
3

6. Reckoning from May, and adding the excesses of the days in the months as they end, we have 3, 5, 8, 11, 13, 16, 18. No one of these making an exact number of weeks, the days of the week do not recur on the same days of the month as in May, in any subsequent month in that year.

REDUCTION.

Pages 188-192.

Example I.

OPERATION.

3 da. 5 hr. 17 min.
24
72
5
77 hr.
60
4620
17
4637 min.
60
278220 sec.

CONVERSE.

6|0)27822|0 sec.
6|0)463|7 min. — 17 min.
24)77 hr. — 5 hr.
3 da.

∴ 278220 sec. = 3 da. 5 hr. 17 min.

2. EXPLANATION. — Since every 2 pt. make 1 qt. in 442 pt., there are as many quarts as 2 is contained times in 442; that is, 221 qt. Again, since every 8 qt. make 1 pk., in 221 qt. there are as many pecks as 8 is contained times in 221; that is, 27 pk. and 5 qt. Finally, since every 4 pk. make 1 bu., in 27 pk. there are as many bushels as 4 is contained times in 27; that is, 6 bu. and 3 pk. Hence 442 pt. = 6 bu. 3 pk. 5 qt.

OPERATION.

2) 442 pt.
8) 221 qt. — 5 qt.
4) 27 pk. — 3 pk.
6 bu.
∴ 442 pt. = 6 bu.
3 pk. 5 qt.

Converse. Since there are 4 pk. in a bushel, in 6 bu. there are 4 times 6, or 24 pk.; to which adding the 3 pk., we have 27 pk. Again, since 8 qt. make a peck, in 27 pk. there are 8 times 27, or 216 qt.; to which adding the 5 qt., we have 221 qt. Finally, since 2 pt. make a quart, in 221 pt. there are 2 times 221, or 442 pt. Hence 6 bu. 3 pk. 5 qt. = 442 pt.

6 bu. 3 pk. 5 qt.
4
27 pk.
8
221 qt.
2
442 pt.

[In like manner, each of the 99 examples in this set should be made to furnish two exercises, and the demonstrations should be repeated till they can be given with readiness.]

3. 6 gal. 3 qt. 1 pt.
4
27 qt.
2
55 pt.

2) 55 pt. — 1 pt.
4) 27 qt. — 3 qt.
6 gal. 3 qt. 1 pt.

4. £32 12*s.* 10*d.*
20
652*s.*
12
7834*d.*

12) 7834*d.* — 10*d.*
20) 652 2*s.* — 12*s.*
£32 12*s.* 10*d.*

5. 8 lb. 6 oz. 12 pwt. 16 gr.

12
102 oz.
20
2052 pwt.
24
8224
4104
49264 gr.

Gr.
49264 (24
48 2|0)205|2 pwt. — 12 pwt.
126 12)102 oz. — 6 oz.
120 8 lb. 6 oz. 12 pwt.
64 16 gr.
48
16 gr.

12. $4\frac{3}{4}$ cords.

128 cu. ft. in 1 cord.
64
32
512
608 cu. ft.

Cu. Ft.
608(128
512 $4\frac{3}{4}$ cords.
8) 96 = 4)12 = $\frac{3}{4}$
128 16

18. $\frac{2}{3}$ gal. $= \frac{2 \times 4 \times 2}{3} = \frac{16}{3} = 5\frac{1}{3}$ pt.

Conversely. $5\frac{1}{3}$ pt. $= \frac{\overset{2}{\not 16}}{3 \times \not 4 \times \not 2} = \frac{2}{3}$ gal.

30. 16 lb. 8 oz. = 16.5 lb. Then, since 60 lb. make a bushel, 16.5 lb. make $\frac{16.5}{60} = .275$ bu. Or, $\frac{16.5}{60} = \frac{165}{600} = \frac{33}{120} = \frac{11}{40}$.

Conversely. Since 60 lb. make a bushel of wheat, .275 bu. is 60 times .275, or 16.5 lb., or 16 lb. 8 oz.

33. $\frac{4}{5}$ bu. $= \frac{4 \times 4}{5} = \frac{16}{5} = 3\frac{1}{5}$ pk.

$\frac{1}{5}$ pk. $= \frac{1 \times 8}{5} = \frac{8}{5} = 1\frac{3}{5}$ qt.

$\frac{3}{5}$ qt. $= \frac{3 \times 2}{5} = \frac{6}{5} = 1\frac{1}{5}$ pt.

$\therefore$ $\frac{4}{5}$ bu. = 3 pk. 1 qt. $1\frac{1}{5}$ pt.

Conversely. $1\frac{1}{5}$ pt. $= \frac{6}{5}$ pt. $= \frac{6}{5 \times 2} = \frac{3}{5}$ qt.

$\therefore$ 1 qt. $1\frac{1}{5}$ pt. $= 1$ qt. $+ \frac{3}{5}$ qt. $= 1\frac{3}{5}$, or $\frac{8}{5}$ qt.

$\frac{8}{5}$ qt. $= \frac{8}{5 \times 8} = \frac{1}{5}$ pk.

$\therefore$ 3 pk. 1 $\frac{3}{5}$ qt. $= 3$ pk. $+ \frac{1}{5}$ pk. $= 3\frac{1}{5} = \frac{16}{5}$ pk.

$\frac{16}{5}$ pk. $= \frac{\overset{4}{\cancel{16}}}{5 \times \cancel{4}} = \frac{4}{5}$ bu.

A neat *form* for such solutions is the following: —

$$2)1\tfrac{1}{5} = \tfrac{6}{5}$$
$$8)1 \quad + \tfrac{3}{5} = \tfrac{8}{5}$$
$$4)3 \quad + \tfrac{1}{5} = \tfrac{16}{5}$$
$$\tfrac{4}{5}$$

It will be observed that this is the form most convenient to use when we reduce a compound number from lower to the decimal of a higher denomination. See next solution.

36. In 8 gr. are $\frac{1}{20}$ as many scruples, or .4 scruples, which added to 2 scruples make 2.4 scruples. In 2.4 scruples are $\frac{1}{3}$ as many drams, or .8 drams, which added to 1 dram makes 1.8 drams, etc.

2.0) .8
3) 2.4
8) 1.8
12) 9.225
.76875

51. Since 60 ℥ = 1 ƒ℥, 36 ℥ = $\frac{36}{60}$ = .6 ƒ℥, which added to 5 ƒ℥ makes 5.6 ƒ℥. Since 8 ƒ℥ = 1 ƒ$\overset{z}{\text{ʒ}}$, 5.6 ƒ℥ $= \frac{5.6}{8}$, or .7 ƒ$\overset{z}{\text{ʒ}}$. (See **231.**)

6.0) 36
8) 5.6
.7

55. Since 1 fr. = \$0.193, 1250 fr. = \$0.193 × 1250 = \$241.25.

Conversely. \$241.25 $= \frac{241.25}{.193} = 1250$ fr.

56. 1 Nap. = 20 fr. $\therefore$ 5 Nap. = 5 × 20, or 100 fr. 1 fr. = \$0.193. $\therefore$ 100 fr. = \$0.193 × 100 = \$19.30.

Conversely. Since 1 fr. = \$0.193, \$19.30 $= \frac{19.30}{.193} =$

100 fr. And, as 20 fr. make 1 Nap., 100 fr. $= \frac{100}{20} = 5$ Nap.

62. A lot 5×8 rd. contains $5 \times 8 = 40$ sq. rd. Now 1 sq. rd. is $\frac{1}{160}$ of an acre: hence 40 sq. rd. $= \frac{40}{160} = \frac{1}{4}$ acre.

64. 7.21 ch. by 3.40 ch. $= 7.21 \times 3.4 = 24.514$ sq. ch. But 10 sq. ch. $= 1$ A. $\therefore$ 24.514 sq. ch. $= 2.4514$ A.

74. $\frac{1}{12}$ bu. $= \frac{1 \times 4 \times 8 \times 2}{12}$ pt. $= \frac{16}{3}$ pt. $= 5\frac{1}{3}$ pt.

Any number of bushels makes 4 times as many pecks, any number of pecks 8 times as many quarts, and any number of quarts makes 2 times as many pints.

82. 3 lb. 10 oz. $= 3 \times 12 + 10$, or 46 oz. 20 gr. $= \frac{20}{24 \times 20}$ oz. $= \frac{1}{24}$ oz. Hence 3 lb. 10 oz. 20 gr. $= 46\frac{1}{24}$ oz.

Conversely. How many lb., oz., and gr. in $46\frac{1}{24}$ oz.? 46 oz. $= \frac{46}{12}$ lb. $= 3$ lb. 10 oz.; and $\frac{1}{24}$ oz. $= \frac{1 \times 20 \times 24}{24}$ gr. $= 20$ gr. Hence $46\frac{1}{24}$ oz. $= 3$ lb. 10 oz. 20 gr.

83. 342 oz. $= \frac{342}{12}$ lb. $= 28$ lb. 6 oz.; and $\frac{2}{5}$ oz. $= \frac{20 \times 2}{5}$ pwt. $= 8$ pwt. Hence $342\frac{2}{5}$ oz. $= 28$ lb. 6 oz. 8 pwt.

Conversely. 28 lb. $= 28 \times 12$ oz. $= 336$ oz. 8 pwt. $= \frac{8}{20}$ oz. $= \frac{2}{5}$ oz. Hence 28 lb. 6 oz. 8 pwt. $= 336$ oz. $+ 6$ oz. $+ \frac{2}{5}$ oz. $= 342\frac{2}{5}$ oz.

Or thus,

28 lb. $= 28 \times 12$ oz. $= 336$ oz.
6 oz. $= 6$ oz.
8 pwt. $= \frac{8}{20}$ oz. $= \frac{2}{5}$ oz.
$342\frac{2}{5}$ oz.

84.

1 T. 13 cwt. 58 lb.
20
33
100
3358

1|00) 33|58 — 58 lb.
2|0) 3|3 — 13 cwt.
1 T.

85. 100)7129 lb.
2.0)7.1 cwt. 29 lb.
3 T. 11 cwt. 29 lb.

Conversely.
20
71 cwt.
100
7129 lb.

85. 7129 lb. (112
672 2.0) 6.3 cwt. 73 lb.
409 3 T. 3 cwt. 73 lb.
336 20 *Conversely.*
73 lb. 63 cwt.
112
199
693
7129 lb.

92. 3.0)1.3 da.
12)2.4333 mo.+
0.2028 yr.—

The converse will not come out exact unless the fractions are exact. Thus we have,

3|0)1|3 da.
12)2.4$\frac{1}{3}$ mo.
0.2$\frac{1}{36}$ yr.
12 *Conversely.*
2.4$\frac{1}{3}$ mo.
30
2 mo. 13 da.

NOTE. — In reducing time to decimals of years for computing interest at 30 da. to a month, by a little care the entire fraction may be saved, and yet not involve a burdensome common fraction. Thus, in the above, while .2$\frac{1}{36}$ yr. would not be convenient as a multiplier,

if the decimal is extended a little, we obtain .202$\frac{7}{9}$ yr., and in most cases the common fraction can be put in still lower terms.

93. 3|0)|7 da.

12)5.2333+ mo.

2.43611+, or 2.436$\frac{1}{3}$ yr.

Conversely.

12

5.233$\frac{1}{3}$ mo.

30

7.000 da.

∴ 2.436$\frac{1}{3}$ yr. = 2 yr. 5 mo. 7 da.

95. 3|0) 2|4 da.

12)10.8 mo. + (4 × 12) mo. = 58.8 mo.

4

99. 3|0)2|4 da.

12)4.8 mo.

1.4 yr.

COMPOUND ADDITION.

[☞ Do not allow pupils to write compound numbers thus:

£ *s.* *d.*

25 — 16 — 2 nor 5 lb. — 3 oz. — 8 pwt.

Whatever *they* may mean, these are £25 *minus* 16*s.* *minus* 2*d.*, etc. These numbers should be written as they are printed, without any mark between the denominations except the symbol of the denomination. This may be written above the figures, if desired; that is frequently the more elegant and perspicuous writing.]

Pages 193, 194.

Example 3.

EXPLANATION. — We write numbers of the same denomination in the same column, because such are more conveniently added together. We then begin with the lowest denomination, because we can thus tell whether there will arise any of the higher denominations from adding the lower, and, if there does, can add it in with the higher denominations as we go along.

£105	1*s.*	2*d.*	3*fr.*
218	11	5	2
199	17	9	2
77	18	3	3
£601	8*s.*	9*d.*	2*fr.*

In this example the sum of the farthings is $10fr. = 2d.$ $2fr.$ Writing the $2fr.$ in its place, we add the $2d.$ to the pence, which amount to $21d. = 1s.$ $9d.$ Writing the $9d.$ in its own place, we add the $1s.$ to the shillings, and have $48s.$ $= £2$ $8s.$ Writing the $8s.$ in its own place, we add the £2 to the pounds, and have £601. Hence the sum is £601 $8s.$ $9d.$ $2fr.$

4.

8 lb.	11 oz.	6 dr.	2 scr.
9	10	7	1
4	7	3	1
17	8	3	1
45	11	3	1
87 lb.	2 oz.	0 dr.	0 scr.

or 87 lb. ℥ ij.

The first paragraph of the explanation given above is applicable to all examples of this kind.

In this example the sum of the scruples is 6 scr. $= \frac{6}{3}$ dr. $=$ 2 dr. Adding this with the drams, we have 24 dr. $= \frac{24}{8}$ oz. $=$ 3 oz. Adding this with the ounces, we have 50 oz. $=$ $= \frac{50}{12}$ lb. $=$ 4 lb. 2 oz. Writing the 2 oz. under its own denomination and adding the 4 lb. with the pounds, we have 87 lb. Hence the sum is 87 lb. ℥ ij.

5.

10 rd.	3 yd.	1 ft.	7 in.
7	2	2	5
3	4	1	9
5	2	1	10
13	4	0	11
41 rd.	$\frac{1}{2}$ yd.	2 ft.	6 in.

The sum of the inches is 42 in. = 3 ft. 6 in.
The sum of the feet is 8 ft. = 2 yd. 2 ft.
The sum of the yards is 17 yd. = 3 rd. $\frac{1}{2}$ yd.
The sum of the rods is 41 rd.

Now the $\frac{1}{2}$ yd. = $1\frac{1}{2}$ ft., or 1 ft. 6 in. This with the 2 ft. 6 in. makes 4 ft. = 1 yd. 1 ft. Hence the sum is 41 rd. 1 yd. 1 ft.

11.

5 bbl.	30 gal.	3 qt.	$1\frac{1}{4}$ pt.
7	25	2	0
	28	1	0
3	30	0	$1\frac{3}{4}$
		2	$1\frac{1}{2}$
	13	2	$1\frac{3}{4}$
18 bbl.	$24\frac{1}{2}$ gal.	1 qt.	$0\frac{1}{4}$ pt.
Or, as $\frac{1}{2}$ gal. =		2 qt.	
We have 18 bbl.	24 gal.	3 qt.	$\frac{1}{4}$ pt.

12.

$\frac{3}{4}$ bu. = .75 bu.

2.64 pk. = $\frac{2.64}{4}$ bu. = .66 bu.

.5 bu. = .5 bu.

$12\frac{3}{5}$ qt. = $\frac{63}{5}$ qt. = $\frac{63}{5 \times 32}$ bu. = .39375 bu.

$10\frac{1}{2}$ pk. = $\frac{10.5}{4}$ bu. = 2.625 bu.

320 pt. = $\frac{320}{2 \times 8 \times 4}$ bu. = 5.000 bu.

9.92875 bu.

13. In.

12)2005

167.0833+ ft.

332.15 ft.

3)499.2333+ ft.

166.4111+ yd.

16.0166+ yd.

$5\frac{1}{2}$)182.4277+ yd.

11)364.8555+

33.168+ rd.

Ft.	Yd.
2	
75.5	8
28.4	4.35
226.25	3.6666+
332.15 ft.	16.0166+ yd.

NOTE. — Both teacher and pupil should study that work put upon the blackboard, slate, or paper, be in the most neat, compact, and intelligible form. It would be a very easy matter to put this example upon the board so that it would occupy several times as much space, and require several times as many figures, as in the above form, and at the same time be little more than a confused mass of figures. Let this form be carefully studied, and compared with others which can be readily produced.

14. 40 rd. is 10 ch. Hence the last piece is $10 \times 40 =$ 400 sq. ch. $= 40$ A. This with the other 40 A. $+ 80$ A. makes $\frac{1}{2}$ sec. Hence in all he had $1\frac{1}{2} + \frac{1}{2} = 2$ sec.

[This example also affords a good opportunity to observe elegance and conciseness of method. Solved as above, the sum is seen at a glance.]

COMPOUND SUBTRACTION.

Page 196.

7. $\frac{1}{2}$ bu. $= 16$ qt.
3 qt. 2 pt. $=$ 4 qt.
12 qt.
or 1 pk. 4 qt.

13. $2\frac{1}{2}$ cd. $= 2\frac{1}{2} \times 128 = 320$ cu. ft.

120.32
128)199.68
1.56 cd.

The forms of explanation given in the text-book are all that can be needed in this subject.

17.	ʒ	ʒ	℈	18.	31 gal.	2 qt.		
	7	3	1		10 gal.	3 qt.	1 pt.	
	3	5	2		20 gal.	2 qt.	1 pt.	
	3	5	2					
or	ʒ iij	ʒ v	℈ ij					

Page 197.

2. From April 5, 1860, to April 5, 1875, are 15 yr., including 3 Februaries of 29 da. Hence there are $15 \times 365 + 3$ da. $= 5478$ da.

Now from April 5 to Nov. 24 there are $25 + 31 + 30 + 31 + 31 + 30 + 31 + 24$ da. $= 233$ da. Hence in all, from April 5, 1860, to Nov. 24, 1875, there are 5478 da. $+$ 233 da. $= 5711$ da.

3. $27 + 31 + 30 + 31 + 30 + 25 = 174$.

4. $25 + 30 + 31 + 10 = 96$.

Page 198.

2.	1871	12	5
	1850	3	16
	21	8	19
3.	1846	8	23
	1827	4	2
	19	4	21

Page 199.

1875	6	15
1829	10	20
45	7	26

2. The period terminates (**264**) in June; the month next preceding is May, which has 31 da.

4.	1875	8	15	**5.**	1857	7	25	
	1868	7	20		5	3	24	
	7	0	26		1862	11	18	

6. To interpret such a result as this, we are to remember that any particular date, as we name it, indicates 1 yr. 1 mo. more than the time elapsed since the Christian era began; i.e., July 17, 1862, or 1862 7 17, is the 17th day of the 7th month of the 1862d year of the era. But this is 1861 yr. 6 mo. 17 da. after the beginning of the era. Now, interpreting our result above in this way (i.e., taking out 1 yr. 1 mo.), we find the time 1873 yr. 11 mo. 29 da. after the beginning of the era. This brings us 29 da. into the 1874th year, 12th month: hence the *date* is 1874 12 29, or Dec. 29, 1874. See (**269**) of text-book.

1877	6	8	
2	5	10	
1875	0	29	

7.

1876	2	12
10	1	15
1866	0	28

or Dec. 28, 1865.

8.

1875	12	10
1873	9	7
2	3	3

$\therefore 730 + 91 + 3 = 824$ da.

COMPOUND MULTIPLICATION.

Pages 200, 201.

Example 7.

A.	Sq. Rd.	Sq. Yd.	Sq. Ft.
17	35	4	5
			23
51	105	92	9)115 — 7 sq. ft.
34	70		12 sq. yd. + 92 sq. yd. =
391	805		$30\frac{1}{4})104$ — $13\frac{1}{4}$ sq. yd.
			3 sq. rd. + 805 sq. rd. =
			16\|0)80\|8 — 8 sq. rd.
			5 A. + 391 A =
			396 A. 8 sq. rd. 14 sq. yd. 36 sq. in.

In the first instance we get 396 A. 8 sq. rd. $13\frac{1}{4}$ sq. yd. 7 sq. ft. But $\frac{1}{4}$ sq. yd. $= \frac{9}{4}$ sq. ft. $= 2\frac{1}{4}$ sq. ft. Then the 2 sq. ft. with the 7 sq. ft. make 9 sq. ft. $= 1$ sq. yd. This with the 13 sq. yd. makes 14 sq. yd. Finally, the $\frac{1}{4}$ sq. ft. $= \frac{144}{4} = 36$ sq. in.

8.

Mi.	Rd.	Ft.
4	11	17
		250
1000	27500	$16\frac{1}{2})4250$ — $9\frac{1}{2}$ ft.
		257 rd.
		27500
		320)27757
		86
		1000

Ans., 1086 mi. 257 rd. 3 yd. 6 in.

9.

Da.	Hr.	Min.	Sec.
5	15	13	20
			341
216	75	113	682\|0(6\|0
1705	1705	1023	40 sec.
1921 da.	341	341	
	5190(24	454\|6(6\|0	
	48	46 min.	
	39		
	24		
	150		
	144		
	6 hr.		

Hence we have 1921 da. 6 hr. 46 min. 40 sec.

Compound multiplication affords a good opportunity for pupils to exercise their taste and judgment in putting the work in good form. When the multiplier can be resolved

into small factors, it will usually be found most elegant to multiply by the factors. The above suggest convenient forms when the multiplier is large and a prime number.

10. $45 = 3 \times 3 \times 5$.

	10°	12′	14″
			3
Product by 3,	30°	36′	42″
			3
Product by 3×3,	91°	50′	6″
			5
Product by $3 \times 3 \times 5$,	459°	10′	30″

COMPOUND DIVISION.

Page 202.

8. 115) 6 mi.
320
1920 rd. (16 rd.
115
770
690
80 rd.
16½
1320 (11 ft.
115
170
115
55
12
660 (5 17/23 in.
575
5) 85/115 = 17/23 in.

9. 22) 1 bbl.
31½
31½ (1 gal.
22
9½ gal.
4
38 (1 qt.
22
16 qt.
2
32 (1 5/11 pt.
22
10/22 = 5/11 pt.

There is nothing of special difficulty nor of special interest in compound division.

LONGITUDE AND TIME.

Pages 203-206.

1. Fort Kearney is $99° - 84°$, or $15°$ west of Adrian. Hence the hour of the day is 1 hour earlier at Fort Kearney than at Adrian. It is therefore 8 A.M. at the former place when it is 9 A.M. at the latter. When it is noon at Fort Kearney, it is 1 hr. later, or 1 P.M., at Adrian.

2. EXPLANATION. — The difference in longitude is $48\frac{1}{2}°$; and as each degree of longitude makes a difference of 4 min. in time, the difference in time is $68\frac{1}{2} \times 4$, or 194 min. $= 2$ hr. 14 min. Then, when it is 6 A.M. at New York, it is 3 hr. 14 min. earlier in the day at San Francisco, or 2 o'clock 46 min. A.M. Again, it is 3 hr. 14 min. later in the day at New York than at San Francisco. Hence, when it is 4 P.M. at the latter place, it is 4 hr. + 3 hr. 14 min. at the former, or 7 hr. 14 min. P.M.

OPERATION.

$122\frac{1}{2}°$
$74°$
$48\frac{1}{2}°$
4
6|0)194
3 hr. 14 min.
6
2 hr. 46 min.

3. $\frac{38}{51.1} = \frac{380}{511}$. $\frac{380}{511}$ of 4 min. $= 2.97+$ min., or about 3 min.

4. $\frac{1}{2}$ of $15° = 7\frac{1}{2}°$.

5. $3\frac{1}{2} \times 15° = 52\frac{1}{2}°$. $2\frac{1}{2} \times 15' = 37\frac{1}{2}'$. 2 hr. $= 30°$. 15 min. $= \frac{1}{4}$ hr. $= \frac{1}{4}$ of $15° = 3\frac{3}{4}°$. 20 sec. $= \frac{1}{3}$ min. $= \frac{1}{3}$ of $\frac{1}{4}° = \frac{1}{12}°$. Therefore 2 hr. 15 min. 20 sec. $= 30° + 3\frac{3}{4}° + \frac{1}{12}° = 33°\ 50'$. Or 2 hr. 15 min. 20 sec. $= 2\frac{23}{60}$ hr. Then $2\frac{23}{60} \times 15° = 33\frac{5}{6}° = 33°\ 50'$. Or 2 hr. 15 min. 20 sec. $=$ 135 min. 20 sec.

$$4)\underline{135 \text{ min. } 20 \text{ sec.}}\quad 33°\ 50'.$$

See (271, II.)

6. Difference in long., $90° - 74° = 16°$. Hence the difference in time is $\frac{16}{15}$ hr., or 1 hr. 4 min. As New Orleans is west of New York, the hour of the day is earlier; and, when

it is 2 P.M. at New York, it is 2 hr. — 1 hr. 4 min., or 56 min. past noon. Again, when it is 11 A.M. at New Orleans, it is 1 hr. 4 min. later at New York, or 12 o'clock and 4 min.; i.e., 4 min. past noon.

7. 87° 37′ 37″ Longitude of Chicago.
84° 27′ " " Cincinnati.

3° 10′ 37″ Difference in longitude.
4

12° 42′ 28″

12 min. 42 sec. $\frac{28}{60}$ Difference in time,
or 12 min. 42½ sec. nearly.

5 hr. 00 min.
12 42½ sec.

5 hr. 12 min. 42½ sec., time at Cincinnati.

8 hr. 00 min.
12 42½ sec.

7 hr. 47 min. 17½ sec., time at Chicago.

9. The difference in time is 3 hr. 20 min. — 2 hr. 27 min. = 53 min. But each 4 min. time makes a degree of longitude. Hence the difference in longitude is $\frac{53}{4} = 13° 15′$, and the longitude of St. Louis is 77° 0′ 15″ + 13° 15′ = 90° 15′ 15″.

11. EXPLANATION. — As 4 min. of time correspond to 1° of longitude, $\frac{51}{4} = 12.75$ is the difference in longitude between Boston and Ann Arbor. Then, as 51.1 mi. make a degree of longitude at this latitude, the distance is 12.75 × 51.1, or 651.525 mi.

4)51
12.75
51.1
1275
1275
6375
651.525

12. Since one place is in east longitude and the other in west, the difference in longitude is the *sum* of the longitudes, or 84° 28′ 02″, which is reduced to time by multiplying by 4. Hence the difference in time is 5 hr. 37 min. 52 sec. Now, when it is 4 A.M. at Berlin, it is 5 hr. 37

71° 4′ 9″
13 23 53

84° 28′ 02″
4

5 hr. 37 min. 52 sec. 08
4 hr.

1 hr. 37 min. 52 sec.
12 hr.

10 hr. 22 min. 08 sec.

min. 52 sec. earlier in the day at Boston. But 4 hr. earlier brings us to 12 midnight. Hence the time at Boston is 1 hr. 37 min. 52 sec. before 12 midnight, or 10 hr. 22 min. 08 sec. P.M.

13. $\frac{422.413}{53.47} = 7.9$, the number of degrees' difference in longitude. Now 7°.9 corresponds to 7.9×4, or 31.6 min. = 31 min. 36 sec. difference in time.

14. 39 min. corresponds to $\frac{39}{4}$, or 9°.75. Hence $\frac{509}{9.75}$, or 52.2 mi.+, make a degree of longitude at this latitude.

15. As 15 min. corresponds to $\frac{15}{4}$ or $3\frac{3}{4}°$, and a degree is $50\frac{1}{2}$ mi., you would have to go $3\frac{3}{4}$ times $50\frac{1}{2}$ mi., or $189\frac{3}{8}$ mi.

$$\frac{20}{4} \times 50\tfrac{1}{2} = 252\tfrac{1}{2}, \therefore 252\tfrac{1}{2} \text{ mi.}$$

$$\frac{30}{4} \times 50\tfrac{1}{2} = 378\tfrac{3}{4}, \therefore 378\tfrac{3}{4} \text{ mi.}$$

$$\frac{13\frac{1}{2}}{4} \times 50\tfrac{1}{2} = 170\tfrac{7}{16}, \therefore 170\tfrac{7}{16} \text{ mi.}$$

16. As the sun rises at the place eastward before it does at the place westward, it is 19 min. later at Detroit at the same instant than it is at Chicago. Hence my watch will be 19 min. slow of Detroit time.

19 min. in time corresponds to $\frac{19}{4}$ degrees of longitude, or 4° 45′. Hence

82°	58′	
4	45	
87°	43′	Long. of Chicago.

17.

OPERATION.

	83°	50′	48″.3		
	71	7	24 .9		
	12	43	23 .4		
			4		
	50	53	33 .6	$\frac{33.6}{60} = .56$	
3	45	30			
4	36	23.56			

18. When it is noon at 77° W. long., it is midnight 180° therefrom, or at 103° E. long.; i.e., 116° 21′ — 103° = 13° 27′ west of Pekin. 13° 27′ = 53 min. 48 sec. It is therefore 53 min. 48 sec. past 12 o'clock midnight after Jan. 1; i.e., 53 min. 48 sec. A.M., Jan. 2.

19.

3 hr. 4 min. 10 sec.
2 hr. 35 min. 27 sec.

OPERATION.

$7° 10′ 45″ = 7°\frac{43}{240}$.

$7\frac{43}{240} \times 51.1 = 366.85\frac{13}{24}$.

4)28		43	
	7°	10′	45″
	83	50	48 .3
	76°	40′	3″.3

20. 4 hr. 39 min. = 279 min.

4)279		
69°	45′	
122°	26′	15″
192°	11′	15″
180		
12°	11′	15″
180		
167°	48′	45″

21.

90°	15′	10″
75°	9′	23″.4
15	5	46 .6
		4
60	23	6 .4

1 hr. 23′.1$\frac{1}{15}$

ALIQUOT PARTS.

2.

$42 \times .12\frac{1}{2} = \frac{1}{8}$ of $42 = 5\frac{1}{4} = 5.25$.
$42 \times .16\frac{2}{3} = \frac{1}{6}$ of $42 = 7$.
$42 \times .06\frac{1}{4} = \frac{1}{16}$ of $42 = 2\frac{5}{8} = 2.62\frac{1}{2}$.
$42 \times .33\frac{1}{3} = \frac{1}{3}$ of $42 = 14$.
$42 \times .25 = \frac{1}{4}$ of $42 = 10\frac{1}{2} = 10.50$.
$42 \times .20 = \frac{1}{5}$ of $42 = 8\frac{2}{5} = 8.40$.
$42 \times .10 = \frac{1}{10}$ of $42 = 4.2 = 4.20$.
$42 \times .50 = \frac{1}{2}$ of $42 = 21$.

This will afford an illustration of a convenient form of writing the work on the blackboard.

4.

$1.33\frac{1}{3} \times 15 = 1\frac{1}{3} \times 15 = 15 + 5 = 20$.
$1.10 \times 15 = 1\frac{1}{10} \times 15 = 15 + 1.5 = 16.50$.
$1.50 \times 15 = 1\frac{1}{2} \times 15 = 15 + 7.50 = 22.50$.
$1.25 \times 15 = 1\frac{1}{4} \times 15 = 15 + 3.75 = 18.75$.
$1.12\frac{1}{2} \times 15 = 1\frac{1}{8} \times 15 = 15 + 1.87\frac{1}{2} = 16.87\frac{1}{2}$.
$1.20 \times 15 = 1\frac{1}{5} \times 15 = 15 + 3 = 18$.

5.

$1 \div .12\frac{1}{2} = 1 \div \frac{1}{8} = 1 \times 8 = 8$.
$3 \div .12\frac{1}{2} = 3 \div \frac{1}{8} = 3 \times 8 = 24$.
$1.25 \div .12\frac{1}{2} = \frac{5}{4} \div \frac{1}{8} = \frac{5}{4} \times 8 = 10$.

6.

$1 \div .33\frac{1}{3} = 1 \div \frac{1}{3} = 1 \times 3 = 3$.
$2 \div .33\frac{1}{3} = 2 \div \frac{1}{3} = 2 \times 3 = 6$.
$5 \div .33\frac{1}{3} = 5 \div \frac{1}{3} = 5 \times 3 = 15$.

7.

$15 \times 3.20 = 15 \times 3\frac{1}{5} = 45 + 3 = 48$.
$15 \times 2.10 = 15 \times 2\frac{1}{10} = 30 + 1.50 = 31.50$.
$15 \times 4.33\frac{1}{3} = 15 \times 4\frac{1}{3} = 60 + 5 = 65$.

8.

$4.25 \times 25 = \frac{1}{4}$ of $425 = 106.25$.
$3.40 \times 25 = \frac{1}{4}$ of $340 = 85$.
$11.25 \times 25 = \frac{1}{4}$ of $1125 = 281.25$.

9.

$32¢ \times 3\frac{1}{3} = \frac{1}{3}$ of $320¢ = \$1.06\frac{2}{3}$.
$36¢ \times 3\frac{1}{3} = \frac{1}{3}$ of $360¢ = \$1.20$.
$45¢ \times 3\frac{1}{3} = \frac{1}{3}$ of $450¢ = \$1.50$.

10. $40¢ \times 12\frac{1}{2} = \frac{1}{8}$ of $4000¢ = \$5.00$.
$\$1.12 \times 12\frac{1}{2} = \frac{1}{8}$ of $\$112 = \$14.00$.
$16¢ \times 12\frac{1}{2} = \frac{1}{8}$ of $1600¢ = \$2.00$.
$\$2.20 \times 12\frac{1}{2} = \frac{1}{8}$ of $\$220 = \$27.50$.

12. $120 \times 37\frac{1}{2} = \frac{3}{8}$ of $120 = 30 + 15 = 45$.

14.
$\$5.50$
$1.37\frac{1}{2}$
$\$4.12\frac{1}{2}$

Pages 212, 213.

8.

$\$0.28$			$\$0.30$
6			5
$\$1.68$	cost of 6 lb.		$\$1.50$ 5 lb.
.14	" " $\frac{1}{2}$ lb.	= 8 oz.	.15 $\frac{1}{2}$ lb. = 8 oz.
.07	" " $\frac{1}{4}$ lb.	= $\frac{1}{2}$ of $\frac{1}{2}$ lb. = 4 oz.	.07$\frac{1}{2}$ $\frac{1}{4}$ lb. = 4 oz.
$\$1.89$	" " 6 lb. 12 oz.		.03$\frac{3}{4}$ $\frac{1}{8}$ lb. = 2 oz.
			$\$1.76\frac{1}{4}$

9.

$\$0.14$			$\$0.16$
12			8
$\$1.96$	12 lb.		$\$1.28$ 8 lb.
.07	$\frac{1}{2}$ lb.	= 8 oz.	.08 $\frac{1}{2}$ lb. = 8 oz.
.03$\frac{1}{2}$	$\frac{1}{4}$ lb.	= 4 oz.	.02 $\frac{1}{8}$ lb. = 2 oz.
.01$\frac{3}{4}$	$\frac{1}{8}$ lb.	= 2 oz.	.01 $\frac{1}{16}$ lb. = 1 oz.
$\$2.08\frac{1}{4}$	12 lb. 14 oz.		$\$1.39$ 8 lb. 11 oz.

Observe to get the cost of $\frac{1}{4}$ lb. by taking $\frac{1}{2}$ of the cost of $\frac{1}{2}$ lb., of $\frac{1}{8}$ by taking $\frac{1}{2}$ the cost of $\frac{1}{4}$, etc.

The pupil should have sufficient exercise in such processes to make them entirely familiar, and to give him great facility in using them. They are among the most useful in the common affairs of life, being of service many times where the processes of percentage are used once.

10. $1.20
2
$2.40 cost of 2 bu.
.60 " " ½ bu. = 2 pk.
.07½ " " ¼ pk. = ⅛ of 2 pk. = 2 qt.
.03¾ " " ½ of ¼ pk. = 1 qt.
$3.11¼

11.

15	15	15	15	15
3⅕	2.1	4⅓	3⅛	1⅔
45	15	65	46⅞	15
3	30		or 46.87½.	10
48	31.50			25

12. $\frac{\$56 \times 8^{2}}{12_{3}} = \frac{112}{3} = \$37.33\frac{1}{3}$.

$\frac{\$56 \times 24^{2}}{12 \times 30_{15}} = \frac{11.2}{3} = \frac{3.73\frac{1}{3}}{\$41.06\frac{2}{3}}$.

$\frac{\$56^{14} \times 9^{3}}{12_{4}} = \$42.00$.

$\frac{\$56^{28} \times 20}{12_{3} \times 30} = \frac{28}{9} = \frac{3.11\frac{1}{9}}{\$45.11\frac{1}{9}}$.

$\frac{\$56^{14} \times 10}{12_{3}} = \frac{140}{3} = \$46.66\frac{2}{3}$.

$\frac{\$56^{14} \times 18^{2}}{12_{4} \times 30_{10}} = \frac{28}{10} = \frac{2.80}{\$49.46\frac{2}{3}}$.

$\frac{\$56^{7} \times 25^{5}}{12_{3} \times 30_{3}} = \frac{35}{9} = \$3.88\frac{8}{9}$.

These, together with the examples solved in the book, must be quite ample for the purpose of illustrating this elegant method of solving such problems. By consulting (**348**), the teacher will see an important application of this process. *It is the most expeditious of all methods of solving such problems.* The expert writes but a few of the above figures.

METRIC SYSTEM.

Pages 222-224.

1. 23 cm
47
9
38
74
191 cm

2. 237^{m}
0 53
17 3
24 07
278^{m}9

3. 239^{m}47
0 98
19
70 07
329^{m}52

4. 97 cm
39
58 cm

5. 7^{m}35
4 86
2^{m}49

9^{m}08
57
8^{m}51

6. 573^{m}024
426 72
116 738
111 25
58 52
249 936
1536^{m}188

7. 573^{m}024
426 72
146^{m}304

249^{m}936
111 25
138^{m}686

8. 24 cm
3
72 cm

87 mm
9
783 mm
or 7 dm 83

9. 29^{m}43
7
206^{m}01

216^{m}329
8
1730^{m}632

10. 13)78 cm
6 cm

8)168^{m}
21^{m}

11. 8)16^{m}8
2^{m}1

4)436^{m}324
109^{m}081

12. 8)9864^{Km}
1233^{Km}

7)96^{Hm}
13^{Hm}71+

13. 753^{m}
10
7530^{m}
= 753^{Dm}

14. 6)237^{Km}6
39^{Km}6

15. 9)216^{Km}18
24^{Km}02

16. $34.3)325.85 = 9\frac{1}{2}$

308 7

17 15

17 15

17. $4.5)416776.5 = 92617.$

405

117

90

277

270

76

45

315

315

19. $27^{l} = 2700$ cl.

$27^{l} = 2^{Dl}7.$

20. 3954 cl $= 3^{Dl}954.$

3954 cl $= 0^{Hl}3954.$

3954 cl $= 39^{l}54.$

3954 cl $= 395$ dl 4.

21. $697^{Hl} = 69700^{l}.$

$697^{Hl} = 6970000$ cl.

$697^{Hl} = 697000$ dl.

22. 100 cl $= 1^{l}.$

$\therefore$ $3\not{c}$ pr. cl $=$ \$3 pr. l.

23. $25\not{c}$ pr. l $= .25\not{c}$ pr. cl.

$\therefore$ 2 cl cost $2 \times .25\not{c} = \frac{1}{2}\not{c}.$

24. A liter $= .1^{Dl}.$

$\therefore$ \$1.63 $\times$.1 $= 16.3\not{c}.$

25. $1^{l} = .01^{Hl}.$

$\therefore$ \$2.24 $\times .01 = 2.24\not{c}$, or about $2\frac{1}{4}\not{c}.$

26. \$1.35 $\times$.1 $= 13\frac{1}{2}\not{c}.$

27.

173

2 dl

346 dl

7

2422 dl

or $2^{Hl}422$

2.15

12110

2422

4844

5.20730

29. $27^{T}3 = 27300^{Kg}.$

$27300^{Kg} = 27300000^{g}.$

30. $4^{Kg}3$
$2\ 0$
$\quad 5$
$3\ 5$
$\quad 1$
$\quad 05$
$\quad 5$
$1\ 5$
$12^{Kg}45$

31. $96^{Kg}16$
$\quad 136$
57696
28848
9616
$13077^{Kg}76$
or $13^{T}07776$
$\quad 7\ 832$
$20^{T}90976$ Sum.
$5^{T}24576$ Diff.

32. $7^{T}3 = 7300^{Kg}$.

$\$1.15$
$\quad 7300$
$\overline{845}$
$\quad 805$
$\overline{\$8395}$

33. $1^{T} = 1000^{Kg}$.
$\$265 \div 1000 = 26\frac{1}{2}¢$.

34. $29^{Kg} = 29500^{g}$.
$29500 \times .16 = 4720$.

35. 27^{l} of water $= 27^{Kg}$, since $1^{l} = 1^{Kg}$.

36. 36 cl water $= .36^{l} = .36^{Kg} = 36^{Dg}$.

37. $3^{l}5 = 3^{Kg}5 = 3500^{g}$.

38. $1^{l}93 = 1^{Kg}93 = 1930^{g}$.

39. $173^{l} = 173^{Kg}$.

40. $136^{l} + 125^{l} = 261^{Kg}$.

41. $\quad 1.03$
$\quad\quad 8^{Kg}$
$8^{Kg}24$

42. 42^{l} of milk *should* weigh 1.03×42^{Kg}. $43^{Kg}26$, if it were pure.

43. $1^{Kg}29 - 520^{g} = 1^{Kg}29 - 0^{Kg}52 = 0^{Kg}77 = 0^{l}77 =$ 77 cl.

44. $2^{Kg}225 - 540^{g} = 2^{Kg}225 - 0^{Kg}54 = 1^{Kg}685 = 1^{l}685$.

45. $64^{g}5 - 19^{g} = 45^{g}5 = 0^{Kg}0455 = 0^{l}0455 =$ $45\frac{1}{2}$ ml.

46. $103^{g}5 - 19^{g} = 84^{g}5$.
$84.5 \div 45.5 = 1.86 -$.

PERCENTAGE.

Pages 225-227.

3. As 6 per cent means 6 on a hundred, he lost 6 trees out of a hundred. Now 1 out of a hundred is .01 of them, and 6 out of a hundred is .06 of them. Hence he lost .06 of 1250 trees, or $125 \times .06$; that is, 75 trees.

$\quad 1250$
$\quad\ .06$
75.00

[It is of the first importance that the idea be fixed in mind that 6 per cent of any thing is .06 of it; 7 per cent .07 of it, etc.; and that .06 of any number is found by multiplying the number by .06, etc. *This is the very first and most fundamentally important thing to be taught in percentage.*]

4. The damage being 15 per cent means 15¢ on 100¢, or \$15 on \$100; that is, .15 of the value, as 15 is .15 of 100. Hence the property was damaged .15 of its value. .15 of \$6530 is \$6530 × .15 = \$979.50.

\$6530
.15
32650
6530
\$979.50

5. Making 4 per cent means making 4 on a hundred (as 4¢ on 100¢, or \$4 on \$100). Now, 4 on 100 is .04 of the amount. Hence the speculator made .04 of \$450. .04 of \$450 is \$450 × .04 = \$18.

\$450
.04
\$18.00

7. Since 50 per cent of any thing is .50 of it, and .50 = $\frac{1}{2}$, the flock had increased $\frac{1}{2}$ of its number; i.e., $\frac{1}{2}$ of 340, or 170.

The per cent was 50, the *rate* of increase .50 or $\frac{1}{2}$. The increase was 170.

[Too much pains can scarcely be taken to have such simple exercises as the above clearly understood at the outset. All the examples in the book, and scores or hundreds of others, may be needed for the purpose.]

13-29. ANSWERS. $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{2}{5}$, $\frac{11}{100}$, .15 = $\frac{7}{50}$, .07 or $\frac{7}{100}$, .06 or $\frac{3}{50}$, .04 or $\frac{1}{25}$, .00$\frac{1}{2}$ = $\frac{5}{1000}$ = $\frac{1}{200}$, .00$\frac{3}{4}$ = .0075 = $\frac{75}{10000}$ = $\frac{3}{400}$, .00$\frac{5}{8}$ = .00625 = $\frac{625}{100000}$ = $\frac{1}{16}$, 100 hundredths = once, 200 hundredths = 2 times, 1.50 = 1$\frac{1}{2}$ time.

30-40. ANSWERS. .00$\frac{3}{5}$ or .006, .00$\frac{3}{2}$ or .0015, .00$\frac{1}{3}$, .01$\frac{1}{2}$ or .015, .00$\frac{3}{4}$ or .0075, .00$\frac{5}{8}$ or .00625, .00$\frac{4}{5}$ or .008, .00$\frac{5}{6}$ or .008$\frac{1}{3}$, .01$\frac{1}{4}$ or .0125, .12$\frac{1}{2}$ or .125, .08$\frac{3}{4}$ or .0875.

41-48. ANSWERS. 5, 10, 112, 3, $\frac{7}{10}$, $\frac{1}{2}$, $\frac{1}{4}$.

Pages 232-234.

There can be no need of a key for the elementary exercises in Article 309. Two things should be borne in mind in using them in the class-room: —

1st. They should be made a means of securing an intelligent grasp of the principles involved, and ability to give in good style appropriate explanations, or so-called analyses.

2d. A high value should be set on brief and workmanlike forms of solution. See the hints at the bottom of the page in the text-book, and the note at the bottom of p. 231.

Article 310.

51. \$20 is $\frac{1}{3}$ of \$60. $\frac{1}{3}$ of 100 is $33\frac{1}{3}$. ∴ \$20 is $33\frac{1}{3}\%$ of \$60.

52. 11 is $\frac{1}{5}$ of 55. $\frac{1}{5}$ of 100 is 20. ∴ 11 is 20% of 55.

56. $\frac{1}{3}$ is $\frac{2}{3}$ of $\frac{1}{2}$. $\frac{2}{3}$ of 100 is $66\frac{2}{3}$. ∴ $\frac{1}{3}$ is $66\frac{2}{3}\%$ of $\frac{1}{2}$.

57. $\frac{1}{2}$ is $\frac{3}{2}$ of $\frac{1}{3}$. $\frac{3}{2}$ of 100 is 150. ∴ $\frac{1}{2}$ is 150% of $\frac{1}{3}$.

70. 1¢ is $\frac{1}{100}$ of \$1. $\frac{1}{100}$ of 100 is 1. ∴ 1¢ is 1% of \$1.

\$1 is 100 times 1¢. 100 times 100 is 10000. ∴ \$1 is 10000% of 1¢.

Article 311.

72. 10% is $\frac{1}{10}$. If 245 is $\frac{1}{10}$, 10 tenths is 2450.

73. If 125 is 15%, 1% (or $\frac{1}{100}$) is $\frac{1}{15}$ of $\frac{25}{125} = 8.33\frac{1}{3}$, and 100% (or $\frac{100}{100}$) is 100 times $8.33\frac{1}{3} = 833\frac{1}{3}$.

74. $33\frac{1}{3}\%$ is $\frac{1}{3}$. If 7.15 is $\frac{1}{3}$, 3 thirds is 3 times 7.15, or 21.45.

80. $37\frac{1}{2}\%$ is $\frac{3}{8}$ of a number. If 15 is $\frac{3}{8}$, $\frac{1}{8}$ is 5, and $\frac{8}{8}$ is 40.

85. If $\frac{1}{2}$ is 150%, 1% is $\frac{1}{150}$ of $\frac{1}{2}$, or $\frac{1}{300}$. If $\frac{1}{300}$ is 1% (i.e., $\frac{1}{100}$), 100% (i.e., $\frac{100}{100}$) is 100 times $\frac{1}{300}$, or $\frac{100}{300} = \frac{1}{3}$.

Or, 150% is $1\frac{1}{2}$ times a number, or $\frac{3}{2}$ times. If $\frac{1}{2}$ is $\frac{3}{2}$ of a certain number, $\frac{1}{2}$ of that number is $\frac{1}{3}$ of $\frac{1}{2}$, and $\frac{2}{2}$ is 2 times $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{3}$.

90. If 5 is 8%, 1% is $\frac{1}{8}$ of 5, or $\frac{5}{8}$. If $\frac{5}{8}$ is 1%, 100% is 100 times $\frac{5}{8} = \frac{500}{8} = 62.5$.

Article 312.

93. Since the amount is the number itself + the percentage, the *amount* at 5% is 105% the number. Then, as in the last, if \$61.53 is 105%, 1% is $\frac{1}{105}$ of \$61.53, and 100% is $\frac{100}{105}$ of $\$61.53 = \frac{\$6153}{105} = \$58.60$.

94. \$36.54 is $101\frac{1}{2}\%$ of some number. What is that number? $\frac{100}{101\frac{1}{2}}$, or $\frac{1000}{1015}$ of $\$36.54 = \frac{\$36540}{1015} = \$36$.

109. $\frac{3}{4}$ is 100% + $12\frac{1}{2}\%$; i.e., $1\frac{1}{8} :: \frac{9}{8}$ a certain number. What is that number? $\frac{1}{8}$ is $\frac{1}{9}$ of $\frac{3}{4}$, and $\frac{8}{8}$ is $\frac{\cancel{9}^{3}}{\cancel{8}} \times \frac{1}{\cancel{9}_{3}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{2}{3}$.

Article 313.

111. The "difference" is the number itself (i.e., 100%) minus the percentage. 100% — 7% is 93%. Now, if \$483.60 is 93%, 1% is $\frac{1}{93}$ of \$483.60, and 100% is 100 times $\frac{1}{93}$ of $\$483.60 = \frac{\$48360}{93} = \$520$.

117. 46 is 100% — $3\frac{1}{2}\%$, or $96\frac{1}{2}\%$; i.e., $\frac{965}{1000}$. Then the number is $\frac{1000}{965}$ of 46, or 47.67—.

128. $\frac{3}{5}$ is 100% — $\frac{7}{8}\%$; i.e., $99\frac{1}{8}\% = \frac{99\frac{1}{8}}{100}$ of a certain number. That number is therefore $\frac{\overset{20}{\cancel{100}}}{\underset{793}{\cancel{99}\frac{\cancel{7}}{\cancel{8}}}}$ of $\frac{\overset{24}{\cancel{3}}}{\cancel{5}} = \frac{480}{793}$.

PROFIT AND LOSS.

Pages 235-239.

1. 24% of 350 is $350 \times .24 = 84$. $350 + 84 = 434$. $350 - 84 = 266$.

2. Cost of butter, $12 \times 35 \times 19¢ = \$79.80$. 2% of $\$79.80 = \$1.596$. $\$79.80 + \$1.596 = \$81.396$.

3. 15% of $\$5500 = \$825$.

4. They were worth $1\frac{5}{8}$, or $\frac{13}{8}$ as much as I paid. $\frac{13}{8} \times 3560 = 13 \times 445 = \$5785$.

5. A loss of 25% is a loss of .25 or $\frac{1}{4}$. Hence I had remaining $\frac{3}{4}$ of 20, or 15 horses. 20 horses, at \$225 each, cost \$4500; and 15, at the same, cost \$3375. Now, I made 30 per cent on the cost of the 15 I sold; i.e., 30%, or .30 of \$3375. Hence I made \$1012.50, and sold the 15 horses for \$4387.50. As the horses cost me \$4500, I lost $\$4500 - \$4387.50 = \$112.50$.

\$225	\$225
15	20
\$3375	\$4500
.30	4387.50
\$1012.50	\$112.50
3375	
\$4387.50	

6. As 10% is .10, or $\frac{1}{10}$, a profit of 10% requires that the selling price be $\frac{1}{10}$ more than the cost. $\$4 + \frac{1}{10}$ of $\$4 = \$4.40$.

As $12\frac{1}{2}\%$ is $.12\frac{1}{2}$, or $\frac{1}{8}$, a profit of $12\frac{1}{2}\%$ requires that the selling price be $\frac{1}{8}$ more than the cost. $\$4 + \frac{1}{8}$ of $\$4 = \$4\frac{1}{2}$.

As 25% is .25, or $\frac{1}{4}$, a profit of 25% requires that the selling price be $\frac{1}{4}$ more than the cost. $\$4 + \frac{1}{4}$ of $\$4 = \$5$.

7. Analysis as above. The operations are as follows:

$6¢ + .08$ of $6¢ = 6¢ + .48¢ = 6.48¢$.
$6¢ + .00\frac{5}{8}$ of $6¢ = 6¢ + .03\frac{3}{4}¢ = 6.03\frac{3}{4}¢$.
$6¢ + .01$ of $6¢ = 6¢ + .06¢ = 6.06¢$, etc.

These results are given in cents. In dollars they are:

$.06 + .08$ of $.06 = .06 + .0048 = .0648$.
$.06 + .00\frac{5}{8}$ of $.06 = .06 + .00625 \times .06$
$= .06 + .0003750 = .060375$.
$.06 + .01$ of $.06 = .06 + .0006 = .0606$, etc.

The written forms, if they are put on the board, or on paper, are as follows:

.06	.06	.06	.06
.02½	.00½	.01¼	.05
.0015	.0003	.00105	.0030

or,

.025	.06	.0175
.06	.005	.06
.00150	.00030	.001050

To each of which the cost is to be added.

Either form should be made perfectly familiar, and the pupil should be trained to discern the most convenient one.

8.	5)3.00	2)3.00	30
	.60	1.50	3
	3.60	10	$90
	20	$15.00	87
	$72.00	72.00	$3
		$87.00	

The total cost was $3 × 30 = $90. A loss of 50 per cent on cost made the selling price 50 per cent, or ½ the cost; i.e., $1.50 per yd. 10 yd. at $1.50 per yd. is $15.00. Again: a gain of 20% is a gain of ⅕ of the cost, and, as 1 yd. cost $3.00, a gain of ⅕ of this would make the selling price $3.60. 20 yd. at $3.60 per yd. is $72.00. Hence the whole was sold for $15 + $72 = $87. And there was a total loss of $90 — $87 = $3.

12. Buying at $27 per A. and selling at $36, I make $36 — $27 = $9 on every $27 I spend. Hence on $1 I make $\frac{1}{27}$ of $9 = $\frac{1}{3}$, and on 100 I make ⅓ of 100, or 33⅓. Hence I make 33⅓%.

More briefly: Buying at $27 and selling at $30, I make $3 on $27, ⅓ on 1, or $\frac{100}{9} = 11\frac{1}{9}$ on 100; i.e., 11⅑%.

Buying at $27 and selling at $90, I make $90 — $27 = $63

on \$27; $\frac{1}{27}$ of $63 = \frac{63}{27} = \frac{7}{3}$ on 1, or $\frac{700}{3} = 233\frac{1}{3}$ on 100; i.e., $233\frac{1}{3}\%$.

13. A loss of 75 on 300 is $\frac{1}{4}$ of $75 = 25$ on 100. Hence, as the loss on 100 is the loss per cent, he lost 25%.

If 25 out of a hundred were lost, $100 - 25 = 75$ were left, and 75 on 100 is 75%.

20.

2560	2560	2)25.60 cwt.	
.45	.31	12.80	
12800	2560	\$38.40	Freight.
10240	7680		
\$1152.00	\$793.60	First cost.	
833.00	38.40	Freight.	
\$319.00 Gain.	1.00	Cartage.	
	\$833.00	Total cost.	

If I gain \$319 by spending \$833, I gain $\frac{1}{833}$ of $319 = \frac{319}{833} = .38+$ on \$1, or $38+$ on 100; i.e., 38%.

21. \$5.50 is the cost $+ 10\%$, i.e., it is $\frac{10}{100} = \frac{1}{10}$ more than the cost, or $1\frac{1}{10}$ times the cost. Now, if \$5.50 is $1\frac{1}{10}$, or $\frac{11}{10}$ of the cost, $\frac{1}{10}$ of the cost is $\frac{1}{11}$ of \$5.50, or $50¢$, and $\frac{10}{10}$ is 10 times $50¢$, or \$5.00.

22. \$230 is the cost $- 8\%$; i.e., it is $\frac{8}{100} = \frac{2}{25}$ less than the cost, or $\frac{25}{25} - \frac{2}{25} = \frac{23}{25}$ of the cost. Now, if \$230 is $\frac{23}{25}$ of the cost, $\frac{1}{25}$ of the cost is $\frac{1}{23}$ of \$230, or \$10, and $\frac{25}{25}$ is 25 times \$10, or \$250.

25. What cost 100 was at first marked at 115. To this was then added 5%, or $\frac{1}{20}$ of $115 = 5.75$. Whence what cost 100 was finally marked $115 + 5.75 = 120.75$. Thus there was an advance of $120.75 - 100 = 20.75$ on 100, which is 20.75, or $20\frac{3}{4}$ per cent.

26. $33\frac{1}{3}\%$ is $\frac{1}{3}$. If I fall $\frac{1}{3}$, the remainder is $\frac{2}{3}$. But I wish to sell at 20%, or $\frac{1}{5}$ advance on \$3.20. \$3.20 $+ \frac{1}{5}$ of \$3.20 = \$3.84. If \$3.84 is $\frac{2}{3}$ of my asking price, $\frac{1}{3}$ is \$1.92, and $\frac{3}{3}$ is $3 \times$ \$1.92, or \$5.76.

27. The nails are sold at \$5 per keg. But, as I sell at 15% advance on cost, what cost 100 I sell at 115. Hence \$5 is $\frac{115}{100}$ of cost, and cost is $5 \div \frac{115}{100} = 5 \times \frac{100}{115} = \frac{500}{115} = \$4.35-$.

28. An advance of 25% is an advance of $\frac{1}{4}$ the price. Hence the goods are marked at $\$2.50 + \frac{1}{4}$ of $\$2.50 = \$3.125$ per yd. 20% on cost is $\frac{1}{5}$ of it. Hence an advance of 20% on \$2.50 is $\$2.50 + \frac{1}{5}$ of $\$2.50 = \$3.00$. Then I can fall $\$3.125 - \$3.00 = \$0.125$ on the marked price, and still make 20% on cost. Falling .125 on 3.125 is falling $\frac{.125}{3.125}$ on 1, or $\frac{12.5}{3.125} = 4$ on 100, or 4%.

29. If he made 10% on cost, what cost 100 was sold at 110. Hence the selling price was $\frac{110}{100}$, or $\frac{11}{10}$ of cost. If $12\frac{1}{2}¢$ was $\frac{11}{10}$, $\frac{1}{10}$ was $\frac{1}{11}$ of $12\frac{1}{2} = \frac{25}{22}$; and $\frac{10}{10}$ was 10 times $\frac{25}{22} = \frac{125}{11} = 11\frac{4}{11}$. Hence the cost was $11\frac{4}{11}¢$ per lb. At $11\frac{4}{11}¢$ per lb., 200 lb. cost $11\frac{4}{11} \times 200 = \$22.73-$.

30. If I lose 4% (i.e., 4 on a hundred), what cost me 100 I sell for 96. Hence the selling price is $\frac{96}{100}$ of the cost. Now, as 6¢ is $\frac{96}{100}$, $\frac{100}{100}$, or the cost, is $6 \div \frac{96}{100} = \frac{600}{96} = \frac{200}{32} = \frac{25}{4} = 6\frac{1}{4}$. Then, as the nails cost me $6\frac{1}{4}¢$, by selling at 7¢ I make $\frac{3}{4}¢$ on 1 lb. If I make $\frac{3}{4}¢$ on $6\frac{1}{4}¢$, on 1¢ I make $\frac{1}{6\frac{1}{4}}$ of $\frac{3}{4}$, and on 100, $\frac{1}{6\frac{1}{4}}$ of $\frac{3}{4}$ of $100 = \frac{4}{25}$ of $\frac{3}{4}$ of $100¢ = 12$, or 12%.

[The best practical way to solve most such examples is to put the work in form first, and then cancel as much as may be.]

31. $1.00 + .15 = 1.15$. $4.60 \div 1.15 = 4$. $\$4.00 \times 12 = \$48$.

As I sell at 15% advance on cost, what cost me \$1.00 I sell at \$1.15. Hence what I sell at \$4.60 cost me $4.60 \div 1.15$, or 4. ∴ The boots cost $\$4.00 \times 12 = \$48$ per case.

32. I sell the coffee at $\frac{100}{6} = 16\frac{2}{3}$ per lb. As it cost 14, I make $16\frac{2}{3} - 14 = 2\frac{2}{3}$ cents per lb., or I make $2\frac{2}{3}$ on every 14 I spend. Hence on 1 I make $\frac{2\frac{2}{3}}{14}$, and on 100, $\frac{\overset{50}{\cancel{100}} \times 2\frac{2}{3}}{\underset{7}{\cancel{14}}} = \frac{400}{21} = 19\frac{1}{21}$, or $19\frac{1}{21}\%$.

33. $(75 \div 120)\ 100 = 62\frac{1}{2}$. Hence the tea cost $62\frac{1}{2}¢$ per lb. [The analysis of this step is, what cost 100 was sold at 120. Hence what sold at 1 cost $(75 \div 120)$, and what sold at 100 cost $(75 \div 120)\ 100$.]

Selling tea which cost $62\frac{1}{2}$ at $56\frac{1}{4}$ entails a loss of

$$\left(\frac{62\frac{1}{2} - 56\frac{1}{4}}{62\frac{1}{2}}\right) 100 = \frac{\cancel{6}\frac{1}{4}}{\underset{10}{\cancel{62}\frac{1}{2}}} \times 100 = 10 \text{ on } 100, \text{ or } 10\%.$$

In like manner, selling at 50 entails a loss of

$$\left(\frac{62\frac{1}{2} - 50}{62\frac{1}{2}}\right) 100 = \frac{12\frac{1}{2}}{\underset{5}{\cancel{62}\frac{1}{2}}} \times 100 = 20, \text{ or } 20\%.$$

34. He makes \$100 on \$0. Hence his per cent is $\frac{100}{0}$, or infinity.

So, also, if he sells at \$250, his per cent gain is $\frac{250}{0}$, or infinity.

35. $16\frac{2}{3}\%$ is $\frac{16\frac{2}{3}}{100} = \frac{1}{6}$ of any thing. Hence I sold $\frac{1}{\underset{2}{\cancel{6}}}$ of $\frac{\overset{4}{\cancel{8}}}{5} = \frac{1}{10}$ of the factory for \$800, which was at the rate of \$8000 for the whole. Now if this was 10%, or $\frac{1}{10}$ less than the real value, it was $\frac{9}{10}$ of the real value. Hence $\frac{1}{10}$ the real value was $\$\frac{8000}{9}$, and $\frac{10}{10}$ was $\$\frac{80000}{9} = \$8888.89-$.

36. A quart dry measure is 67.2 cu. in., and a quart liquid measure is 57.75 cu. in. liquid measure. Hence the berries would be sold for $\frac{67.2}{57.75} = 1.16\frac{4}{11}$ times the cost; i.e., at a gain of $16\frac{4}{11}$ hundredths, or $16\frac{4}{11}\%$.

Buying by liquid and selling at dry, there would be received $\frac{57.75}{67.2} = .85\frac{15}{16}$ of the cost; or $.14\frac{1}{16}$ hundredths of the cost would be lost, i.e., $14\frac{1}{16}\%$.

37. He sells at \$12 per lb. Apothecaries' Weight. But a pound Avoirdupois is $\frac{7000}{5760}$ lb. Apothecaries'. Hence a pound Av. would sell at $\frac{700\text{¢} \times 12}{576\text{¢}} = \frac{700}{48} = \frac{175}{12} = \$14.58\frac{1}{3}$. The gain was therefore $\$14.58\frac{1}{3} - \$7.00 = \$7.58\frac{1}{3}$ on each \$7, or $\frac{\$7.58\frac{1}{3}}{7} = \$1.08\frac{1}{3}$ on \$1, or $108\frac{1}{3}\%$.

38. If 1 lb. Av. cost \$5, 1 lb. Apothecaries' cost $\frac{576\text{¢} \times \$}{\frac{7\text{¢}\text{¢}\text{¢}}{140}}$ $= \frac{576}{140} = \frac{144}{35} = \$4\frac{4}{35}$. At 50% or $\frac{1}{2}$ advance this becomes $\$6\frac{6}{35}$. If 1 lb. sells for $\$6\frac{6}{35}$, 1 dr. sells for $\frac{1}{96}$ of $\$6\frac{6}{35} = \$0.06\frac{3}{7}$.

Or, since 7000 gr. = 1 lb. Av. and 60 gr. = 1 dr. Apoth., 1 lb. Av. $= \frac{7000}{60} = 116\frac{2}{3}$ dr. Apoth. And, as this sells at $\$5 + \$2.50 = \$7.50$, we have $\$7.50 \div 116\frac{2}{3} = \$0.06\frac{3}{7}$, the selling price of 1 dr.

39. $\$1750 \div 1.15 = \$1521.74$. $\$1750 - \$1521.74 = \$228.26$.

42. To sell at $\frac{4}{5}$ cost is to lose $\frac{1}{5}$ the cost (i.e., 20%), $\frac{1}{5}$ of 100 being 20.

43. If $\frac{4}{5}$ of the selling price equals the buying price, the other $\frac{1}{5}$ is the gain. Thus $\frac{4}{5}$ is cost, and $\frac{1}{5}$ gain. Hence the gain is $\frac{1}{4}$ the cost, or 25%.

44. 5% of \$15 is 75¢. Hence he sold the hektoliter for \$14.25, or a liter for $\frac{1}{100}$ of $\$14.25 = 14\frac{1}{4}$¢.

45. A franc being 19.3¢ and a mark 23.8¢, on 19.3¢ I make $23.8 - 19.3 = 4.5$. Then on 1¢ I make $\frac{1}{19.3}$ of 4.5, and on 100, 100 times $\frac{1}{19.3}$ of $4.5 = \frac{4500}{193} = 23\frac{61}{193}$.

COMMISSION, BROKERAGE, AND BANKRUPTCY.

Pages 240-242.

1. The 4 doz. sold for $\$1.50 \times 12 \times 4 = \$72$. 10% on $\$72 = \$7.20$, the commission. $\$72 - \$7.20 = \$64.80$.

2. The gloves sold at $\$1.50 \times 12 = \$18$ per doz. His commission per doz. was 10% of $\$18 = \$1.80$. But he received \$12.60 commission. Hence he sold $\frac{\$12.60}{\$1.80} = 7$ dozen.

3. $\frac{13.86}{198} \times 100 = 7$.

4. $\frac{\$375}{.05} = 7500$.

5. $\$58 \times 320 = \$18560$. 5% of $\$18560 = \$928$, the commission. $\$18560 - \$928 = \$17632$.

6. His commission being 2%, he charges me 2 on 100 of all he sells: hence he pays me 98 of 100. Hence \$2450 is $\frac{98}{100}$ of what the wheat sold for. If 2450 is $\frac{98}{100}$, $\frac{1}{100}$ is $\frac{2450}{98}$, and $\frac{100}{100}$ is $\frac{245000}{98} = \$\frac{1225000}{49}$. If 2000 bu. sold at this, 1 bu. sold at $\frac{61.25}{49} = \$1.25$.

7. 5% of $\$650.50 = \$32.52\frac{1}{2}$, the commission. $\$650.50 - (23.75 + \$32.52\frac{1}{2}) = \$594.22\frac{1}{2}$.

8. $\$33000 \div 86000 = \$0.38\frac{8}{43}$. Hence he can pay $38\frac{8}{43}¢$ on \$1, or $38\frac{8}{43}\%$, and on \$650, $\$650 \times .38\frac{8}{43} = \$249.42-$.

9. $\$850 \div 1250 = \$0.68$. Hence it pays 68¢ on \$1, or 68%. 68% of \$95000 is \$64600.

10. $\frac{\$57.82\frac{1}{2}}{.04\frac{1}{2}} = \$1285$.

11. $\frac{\$25.50}{02\frac{1}{2} \times 120} = \frac{\$51¢}{6¢} = \$8.50$, amount received per barrel. $\$8\ 50 \times 120 - \$25.50 = \$994.50$.

12. $\frac{\$318}{1.06} = \$300.$

13. $\frac{\$3000}{1.0625} = \$2823.53—.$

14. $12600 \times 1.37 \times .03\frac{1}{2} = 604.17.$

15. $\frac{17.28 \times 100}{640 \times .6} = 4\frac{1}{2}.$

16. $\frac{(70 - 12.85)\ 100}{7.50 \times 127} = 6.$

17. $\frac{441}{1.05} \div 7 = 60.$

18. $160 \times \$6\frac{1}{4} = \$1000$ received for flour. Commission for selling, $\$1000 \times .03 = \$30$. Expended \$970. Commission thereon, $\$970 \times .01\frac{1}{2} = \$14.55$.

Used in buying plaster, $\$970 - (\$14.55 + \$5.45) = \$950$. This bought $\frac{950}{19} = 50$ bbl. plaster.

19. $\frac{5}{8}$ of $\$2500 = \$1562.50$, amount collected. 5% of $\$1562.50 = \$78.12\frac{1}{2}$, agent's commission. $\$1562.50 - \$78.12\frac{1}{2} = \$1484.375$.

20. $\frac{\$8758.25}{1.025} = \$8544.63+.$

21. $\frac{\$5650}{1.03\frac{1}{3}} = \frac{\$16950}{3.1} = \$5467.74.$ $\$5650 - \$5467.74 = \$182.26.$

22. 24000 lbs. at $25¢ = \$6000$. $2\frac{1}{2}\%$ of $\$6000 = \$150$, the commission. $\$6000 - (\$150 + \$3 + \frac{1}{2}$ of $\$240) = \$5727.$

23. Assets $\$3750 + \$750 + 1250 = \$5750$. Deducting 5% and \$350, leaves \$5112.50 available to pay debts.

Liabilities $\$4500 + \$1575 + \$3000 + \$2540 = \$11615.$

$\frac{\$5112.50}{11615} = 44\frac{88}{253}¢$, amount which can be paid on a dollar, or $44\frac{88}{253}\%$.

STOCKS.

Pages 244, 245.

1. $\$49.50 \times 50 = \$2475$.

2. $100 - 84\frac{1}{2} = 15\frac{1}{2}$.

5. $4\frac{1}{2}\%$ of $\$2000 = \$90$. $4\frac{1}{2}\%$ of $\$650000 = \$29250$.

6. $\frac{\$45000}{5000000} = 9$ mills, the amount assessed on \$1, or .9%. .9% of $\$1500 = \$13.50$.

7. $\frac{1}{3}\%$ of $\$4200 = \$14$. $\$84\frac{1}{2} \times 42 + \$14 = \$3563$.

9. An expenditure of $\$116\frac{1}{2} + \$\frac{1}{2} = \$117$ yields \$8. Therefore the rate is $\frac{\$8}{117} = .06\frac{98}{117}$, or $6\frac{98}{117}\%$.

11. The stocks would pay $\frac{\$5\frac{1}{2}}{81\frac{3}{4}} = \frac{\$22}{327} = .06\frac{238}{327}$ on \$1, or $6\frac{238}{327}\%$. Hence the real estate is the better investment.

INSURANCE.

Pages 246-249.

1. $\frac{2}{3}$ of $\$3000 = \$2000$. $\frac{5}{8}\%$ of $\$2000 = \$12\frac{1}{2}$.

2. $\frac{5}{6}$ of $\$12000 = \$10000$. $\frac{1}{2}\%$ of $\$10000 = \$50$.

3. $80¢ \times 60 = \$48$. $\$48 + \$1 + 75¢. = \$49.75$.

4. 3% of $\$4000 = \$120$.

5. $2\frac{3}{4}\%$ of $\$3500 = \$96.25$.

6. $6\frac{1}{2}\%$ on $\$22400 = \$1456$. 4% on $\$38000 = \$1520$. Total premium, $\$1456 + \$1520 = \$2976$. As the loss occurred after the policy on the ship had expired, the insurance company was liable only for \$22400. $\$22400 - \$2976 = \$19424$.

7. $\frac{.10 \times 100}{5000} = \frac{1}{500}$, or $\frac{1}{500}\%$.

8. $\frac{\$5.50}{.01\frac{1}{2}} = \frac{\$1100}{3} = \$366.66\frac{2}{3}$.

9. $\frac{\$146}{.08} = \$1825.$

10. $\frac{\$59.22}{.07} = \$846.$

11. $\frac{129.60 \times 100}{17280} = \frac{12960}{17280} = \frac{3}{4}.$

12. $\frac{4.25 \times 100}{340} = 1\frac{1}{4}.$

13. $\frac{(\$3160 - \$2) \times 4}{3} = \$4210.66\frac{2}{3}.$

14. $8\frac{1}{4}\%$ of $\$52650 = \$4343.62\frac{1}{2}.$

15. $\frac{2}{3}$ of $(\$16500 + \$3675)$ at $1\frac{1}{2}\% = \$201.75.$

18. Each $1 paid by the company is to refund $0.004 on the premium paid, and $1 − $0.004 = $0.996 on the $800 to be covered on the house. Hence the policy must be for $\frac{\$800}{.996} = \$803.21+.$

19.

$\frac{1}{4}\%$ of $\frac{1}{5}$ of	$200000 =	$300.00
$\frac{1}{5}\%$ of $\frac{1}{4}$ of	$200000 =	$300.00
$\frac{2}{3}\%$ of $\frac{1}{10}$ of	$200000 =	$133.33\frac{1}{3}$
Total premium .		$\$733.33\frac{1}{3}$

Co. A's policy covered	$40000
Co. B's " "	50000
Co. C's " "	20000
In all, the policies covered . . .	$110000

The loss is $15000. Hence each $1 of policy is to cover $\frac{\$15000}{110000} = \frac{\$1.50}{11} = \$0.13\frac{7}{11}.$

$\$0.13\frac{7}{11} \times 40000 =	$5454.55−
$\$0.13\frac{7}{11} \times 50000 =	6818.18+
$\$0.13\frac{7}{11} \times 20000 =	2727.27+
In all . . .	$15000.00

21. $(\frac{4}{5}$ of \$15880$) \div (1 - .04) = \$13233.33\frac{1}{3}$.

22. $\frac{\$7500}{1 - .025} = \$7692.31-$.

TAXES AND DUTIES.

Pages 249-253.

2. $\frac{903434.5}{63000000} = .001\frac{1}{2}$, nearly.

$\$2000 \times .001\frac{1}{2} = \$3.00$.

3. $.001\frac{1}{2} + .000\frac{1}{2} + .003 + .002 + .02 = .027$. Hence the total assessment was $2.7\not{c}$ on \$1.

$\$3000 \times .027 = \$81$. $\quad$ $\$3000 \times .003 = \$9$.

$\$600 \times .027 = \$16.20$. $\quad$ $\$600 \times .003 = \$1.80$.

$\$3000 \times .02 = \$60$.

$\$600 \times .02 = \$12$.

4. $\$630000000 \times .000\frac{1}{20} = \$31500$.

5. $\$2500000000 \times .00001 = \$25000$.

6. Of each \$1 levied, $6\not{c}$ will not be collected, leaving $94\not{c}$ collectible. From this, 5%, or $4.7\not{c}$, must be deducted for collector's fees. Hence there remains \$0.94 — \$0.047 = \$0.893 to apply on the \$5000, and there must be levied \$1 for each \$0.893 in \$5000. $\frac{5000}{.893} = 5599.10$, and the levy must be \$5599.10.

7.

\$5599.10	
.06	
335.9460	Uncollectible.
5599.10	
\$5263.154	Collectible.
.05	
\$263.15770	Collector's commission.
5263.15	
\$5000	Net.

8. $\frac{3}{5}$ of \$250000 = \$150000, the bridge-tax of the larger city.

On each \$1 levied, 10¢ was uncollectible. Hence 90¢ on \$1 was collectible. But collecting this cost $3\frac{1}{2}\%$. $3\frac{1}{2}\%$ of 90¢ is 3.15¢. Hence there remained of each \$1 levied 90¢ — 3.15¢ = 86.85¢ to apply on the \$150000; and there must be levied \$150000 ÷ .8685 = \$17271.57.

As \$15000 is $\frac{3}{160\,00}$ of \$80000000, the man's tax was $\frac{3}{160\,00}$ of \$172711.57 = \$32.38.

9. How much was realized on a levy of \$172711.57, if 10% was uncollectible and $3\frac{1}{2}\%$ paid for collecting? \$172711.57 — \$17271.157 = \$155440.41 was collectible. $3\frac{1}{2}\%$ of this is \$55440.41. Hence the levy netted

\$155440.41 — \$55440.41 = \$150000.

10.

\$2100000	
.001½	
1050000	
2100000	
\$3150.000	Tax levied.
.08	
\$252.00	Uncollectible.
3150.00	
\$2898.00	Collectible.
.03	
86.94	Paid for collecting.
\$2898.00	
\$2811.06	Net.

11. $\frac{\$35.22}{1050}$ is tax on \$1. Hence $\$\frac{425000 \times 1050}{35.22} = \$\frac{425 \times 105}{.003592} =$ \$12670375.75 was the valuation.

The actual value of the city property is $\frac{1500}{1050}$, or $\frac{30}{7}$ of \$12670357.75 = \$54301533.21.

12. $\$\frac{35.22 \times 1500000}{1050} = \$\frac{52800}{7} = \$50314.285+.$

13. $1.00 — $0.045 = $0.955.
$3\frac{1}{2}\%$ of $0.955 is $0.033425.
$0.955 — $0.033425 = $0.921575.
$150000 ÷ .921575 = $162764.83.

15. $21963821.29 ÷ 1355401317 = 1.620466¢ on $1 valuation.

16.

Tax on		Tax on	
$1 =	1.62046¢.	$10 =	16.2046¢.
$2 =	3.24093¢.	$20 =	32,4093¢.
$3 =	4.86139¢.	$30 =	48.6139¢.
$4 =	6.48186¢.	$40 =	64.8186¢.
$5 =	8.10233¢.	$50 =	81.0233¢.
$6 =	9.72279¢.	$60 =	97.2279¢.
$7 =	11.34326¢.	$70 =	$1.134326
$8 =	12.96372¢.	$80 =	$1.296372
$9 =	14.58419¢.	$90 =	$1.458419.

The amounts on $100 to $900, and $1000 to $9000 are readily written from these.

17. Total valuation, $5680 + $1050 = $6730.

From the table we have:		
Tax on $6000,		$97.227
"	700,	11.343
"	30,	.486
Tax on $6730,		$109.06

Total valuation, $548 + $85 = $633.

From the table we have:		
Tax on $600,		$9.7227
"	30,	.4861
"	3,	.0486
Tax on $633,		$10.26

Total valuation, $8572 + $12765 = $21337.

Tax on	$20000,	$324.093
"	1000,	16.2046
"	300,	4.8613
"	30,	.4861
"	7,	.1134
Tax on	$21337,	$345.76

18. He was assessed on $125 ÷ .0162046, or $7713.83. For state purposes, $\frac{5023609.50}{21963821.29}$ of $125, or $28.59. For school purposes, $\frac{999587.91}{21963821.29}$ of $125, or $5.69. Value of his property, $7713.83 ÷ $\frac{2}{3}$ = $11570.74.

19. He was assessed on $\frac{1}{2}$ of $20000, or $10000. His total tax was, therefore, $0.01620466 × 10000 = $162.0466; of which $\frac{9995832191}{2196382129}$, or $7.37, was school tax.

Pages 254, 255.

2. Value of broadcloth, 13*s.* × 650 = 8450*s.*
" carpet, 3*s.* × 1950 = 5850*s.*
" drugget, $2\frac{1}{3}$*s.* × 2560 = $5973\frac{1}{3}$*s.*
Total, $20273\frac{1}{3}$*s.*

30% on $20273\frac{1}{3}$*s.* is 6082*s.*

Value of lace, 2*s.* × 1246 = 2492*s.*
" coach-lace, $1\frac{1}{12}$*s.* × 1200 = 1100*s.*
Total value of laces, 3592*s.*

25% on 3592*s.* is 898*s.*

Hence the total duty is 6082*s.* + 898*s.* = £349 = $4.8665 × 349 = $1698.41.

3. Net weight of a chest 67 lb. — 9 lb. = 58 lb. Total cost, 110 × 58 × 90¢. = $5742. 40% on $5742 = 2296.80, the duty.

4. 50 × 63 gal. = 3150 gal. Less 3% is 3150 gal. × .97 = 3055½. 3055½ × 20¢. = $611.10, the duty.

5. 54 gal. × 15 = 810 gal., the amount of wine. 810 gal. × .98 = 793.8 gal. after deducting leakage.

Specific duty, 20¢. × 793.8 = $158.76
Ad valorem duty, ¼ of 793.8 × 45¢. = 89.30
Total duty, $248.06

6. 80 (1 — .05) = 80 × .95 = 76, the number of dozens subject to duty. $5.25 × 76 = $399, the cost.

Specific duty, 76 × 2¾ × $3 = $627.00
Ad valorem duty, ½ of $399 = 199.50
Total duty, $826.50

7. 1160 lb. × .9 = 1044 lb. net per hhd., and 1044 lb. × 20 = 20880 lb. total net.

Specific duty, 20880 × 3¢. = $626.40.

8.
£34.225 × .35 = £11.97875
£51.1875 × .40 = £20.475
£32.45375, duty.
$4.8665 × 32.45375 = $157.936.

9. .88 fr. × 7563.8 = 6656.144 fr., cost. 40%, or ⅔, of 6656.144 fr. = 2662.4576 fr., the *ad valorem* duty.

$$7583.8 \text{ m.} = \frac{39.37}{36} \text{ of } 7533.8, \text{ or } 8271.85 \text{ yd.}$$

Specific duty, 8271.85 × 8¢. = $661.748
Ad valorem duty, 19.3¢. × 6656.144 = 513.854
Total duty in gold, $1175.602

10. At 112 lb. per cwt., 5 T. 16 cwt. 3 qr. 20 lb. = 13096. This, at $¼ per lb., makes the invoice ¼ of 13096, or $3274. 20%, or ⅕, of 3274 = $654.80, the duty.

SIMPLE INTEREST.

Pages 256-260.

No good purpose can be subserved by solving these examples *in extenso* in this KEY. We shall simply give a few specimens of solutions in addition to those given in the text-book.

10. 1st. Let us solve this in the common way, by taking aliquot parts of the interest for 1 year.

$750.50	Principal.
.07	
$52.5350	Int. for 1 year.
26.2675	" ½ yr., or 6 mo.
8.7558+	" ⅓ of 6 mo., or 2 mo.
2.9186	" ⅒ of 2 mo., or 20 da.
$90.4769	Int. for 1 yr. 6 mo. 20 da.
750.50	
$840.9769+	Amount.

2d. Reducing the time to years, we have $1.7\frac{2}{3}$ yr. Hence,

$$3.0)2.0$$
$$12)6.6\frac{2}{3}$$
$$1.7\frac{2}{3}$$

$750.50	
.07	
$52.5350	Int. for 1 yr.
$1.7\frac{2}{3}$	
5837+	
5837+	
367745	
52535	
$90.4769+	Int. for $1.7\frac{2}{3}$ yr.

24.

	1876	11	4
Subtracting dates makes the	1873	8	6
time	3 yr.	2 mo.	28 da.

By Aliquot Parts.

$$\begin{array}{r} \$81.25 \\ .06 \\ \hline 4.875 \\ 3 \\ \hline 14.625 \\ .8125 \\ .2708 \\ .0541 \\ .0541 \\ \hline \$15.8165 \end{array}$$

By Decimals.

$$\begin{array}{r} 3.0)2.8 \\ 12)2.9\frac{1}{3} \\ \hline 3.2\frac{4}{5} \end{array}$$

$$\begin{array}{r} \$81.25 \\ .06 \\ \hline 4.875 \\ 32\frac{4}{5}. \\ \hline 19500 \\ 2166 \\ 9750 \\ 14625 \\ \hline \$15.8166 \end{array}$$

By Cancellation.

$$4.875 \times 3 = 14.62\frac{1}{2}$$

$$\frac{{}^{2}4.8{}^{4}8 \times {}^{1}2}{{}^{3}12} = .81\frac{1}{3}$$

$$\frac{4.88 \times {}^{7}28}{{}^{3}12 \times 30} = \frac{34.16}{90} = \frac{.38}{\$15.82}$$

Page 264.

21. \$2500 on interest at 10% for 3 yr. amounts to \$3250
\$5500 − \$2500 = \$3000. This on interest at
7% for 3 yr. amounts to 3630
Hence I had on hand \$6880
My new house cost 8560
Hence I had to raise \$1680

22. $\frac{1}{5}$ of \$750 = \$250. $\frac{\$250 \times .06}{12} = \$1.25$.

∴ The interest on the first part was \$1.25; on the second part, 2 × \$1.25 = \$2.50; and on the third part, 3 × \$1.25 = \$3.75. The total amount paid was, therefore, \$750 + \$1.25 + \$2.50 + \$3.75 = \$757.50.

Page 266.

3. From June 10 to Feb. 17 is $20+31+31+30+31+30+31+31+17=252$ da. Hence,

$$\frac{\overset{316}{\$1\$\$0}\times .10\times 252}{\underset{73}{\$6\$}}=\frac{\$7963.2}{73}=\$109.08+.$$

4. From Oct. 1 to May 6 is $30+30+31+31+28+31+30+6=217$ da. Hence,

$$\frac{\$10\$\$\times .0\$\times 217}{\underset{73}{\$6\$}}=\frac{\$2170}{73}=\$29.73-.$$

We will give one more solution under this head, for the purpose of exhibiting exactly the figures which the most economical solution requires.

6.

10	16.42	
31	20.64	
30	6568	
31	9852	
31	3284	
28	338.9088	(73
31	292	$4.64+
30	469	82.10
31	438	$86.74
5	310	
258	292	
	18	

The 16.42 is the 82.10 after 5 has been rejected, which has also been cancelled from 365, giving 73 for the divisor. The 20.64 is $.08\times 258$. Nothing would necessarily be written but the above. If the number of days was taken from a table, nothing would need be written but the figures at the right.

COMPOUND INTEREST.

Pages 271, 272.

8. The amount of any sum for 4 yr. at 8% compound interest, compounded semi-annually, is the same as the amount of the same sum for 8 yr. at 4%, compounded annually.

By the Table.

1.368569
270
9579983
2737138
$369.513630 Amt.
270
$99.51 Interest.

270
.04
10.80
270
280.80
.04
11.2320
280.80
292.032
.04
11.68128
292.032
303.713
.04
12.14852
303.713
315.861

315.861
.04
12.63444
315.861
328.495
.04
13.13980
328.495
341.635
.04
13.66540
341.635
355.30
.04
14.2120
355.30
$369.51 Amount.
270
$99.51 Interest.

9.

250		
12.50		
262.50	1	½ yr.
13.1250		
275.6250	2	½ yr.
13.78125		
289.406	3	½ yr.
14.4703		
303.8763	4	½ yr.
15.19381		
319.0701	5	½ yr.
15.9535		
335.0236	6	½ yr.
16.7512		
$351.7748	7	½ yr. Amount.
250		
$101.77	Interest for 3½ years.	

11.

$500	$500	
50	.10	
550	50	
55.00	3	
605.00	$150	Simple int.
60.50		
$665.50		
500		
$165.50	Compound interest.	
150		
$15.50	Difference.	

12.

$325	
6.50	
331.50	
6.63	
338.13	
6.762	
344.892	
6.8978	
351.79	
7.0358	
358.8258	
7.1765	
366.0023	
7.32	
373.322	
7.466	
380.788	
7.6157	
388.4037	9 qr., or 2 yr. 3 mo.
.02	
7.768	2 mo. 10 da. $= 2\frac{1}{3}$ mo.
7	$= \frac{7}{3}$ of 2 mo.
9)54.376	
6.041	Int. on $388.40 for 2 mo. 10 da.
388.403	
$394.44	

13. The first would be the same as at 5% for 8 years.

By the Table.

$1.477455	$1, 8 yr., at 5%.
540.20	
29549100	
5909820	
7387275	
$798.12119100	

Annually at 10% we have, by computation, as our table does not extend so far,

$540.20	
54.02	
594.22	1 yr.
59.422	
653.642	2 yr.
65.364	
719.006	3 yr.
71.900	
$790.91	4 yr.
798.12	
$7.21	Diff.

14.

$100		
1.00	1%	} 1 quarter.
.50	½%	}
101.50		1st quarter.
1.015		
.5075		
103.0225		2d quarter.
1.0322		
.5161		
104.5708		3d quarter.
1.0457		
.5228		
106.139		4th quarter.
1.0614		
.5306		
107.731		5th quarter.
1.077		
.538		
109.346		6th quarter.
1.093		
.546		
110.985		7th quarter.

110.985	7th quarter.
1.1098	
.5599	
$112.65	8th quarter. Amount.
100	
$12.655	Interest.
14.00	Int. 2 yr., 7%, on $100.
$1.345	Difference.

16.

1877	1	15
1874	8	18
2 yr.	4 mo.	27 da. Time.

$500	
40	
540	
43.20	
583.20	
46.656	
629.856	
50.3885	
$680.2445	For 2 yr.
.08	
3)54.419560	Int. for 6 mo.
18.1398	" 2 mo.
2	
36.2796	" 4 mo.
8.163	" 27 da.
44.44	" 4 mo. 27 da.
680.244	
$724.68	Amt. for 2 yr. 4 mo. 27 da.

ANNUAL INTEREST, ETC.

Page 273.

2.

$525	
.06	
$31.50	1 year's interest.
.06	
$1.8900	1 year's interest on 1 year's interest.
6	
11.34	3 + 2 + 1 = 6 yr. int. on deferred payments of int.
126.00	4 years' annual interest.
525.00	Principal.
$662.34	Amount.

3. At the end of each 6 mo., $13.75 interest would fall due. Of these amounts there would be 7, on the first of which there would accrue . . . 3 yr. 2 mo. 17 da. int.

On the 2d	2 yr. 8 mo. 17 da.	"
" 3d	2 yr. 2 mo. 17 da.	"
" 4th	1 yr. 8 mo. 17 da.	"
" 5th	1 yr. 2 mo. 17 da.	"
" 6th	8 mo. 17 da.	"
" 7th	2 mo. 17 da.	"
In all	11 yr. 11 mo. 29 da.	

Or 1 day less than 12 yr. interest.

Besides this, there is the interest on the principal for the last 2 mo. 17 da.

The computation is as follows: —

$275	13.75	$275		
2)27.50	1.375	6)27.50		
13.75	12	4.583	2 mo.	int.
7	16.500	1.146	½ mo.	"
96.25	.004	.152	2 da.	"
	16.496	275.00		

Amount of $275 for 2 mo. 17 da. . . $280.881

Amount of $275 for 2 mo. 17 da. . . .	$280.881
The 7 payments of annual interest . .	96.25
Interest on the interest	16.496
Amount of note	$393.63

4. $2.00 int. falls due each quarter. On the first quarter s int. there will be 9 mo. int.; on the 2d, 6 mo.; on the 3d, 3 mo.; on the last, none; or 18 mo. $= 1\frac{1}{2}$ yr. in all. $1\frac{1}{2}$ yr. int. on $2, at 8%, is 24¢.

∴ Principal	$100.00
Interest	8.00
Interest on interest . .	.24
	$108.24

5. The *interest upon the interest deferred* is 9 yr. on the first year's interest, 8 yr. on the second, 7 yr. on the third, etc.; that is, $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ years' interest on 1 year's interest, or $20. This is $90. To this add the simple interest, $200, and we have the amount, $\$200 + \$200 + \$90 = \$490$.

6.

$350	$24.50	2 yr. 5 mo. 10 da.
.07	.07	1 yr. 5 mo. 10 da.
24.50	1.715	5 mo. 10 da.
3	$4\frac{1}{3}$	4 yr. 4 mo., or
$73.50	571	$4\frac{3}{4}$ yr. int. on 1 yr. int.
8.1666	6.860	
2.0416	$7.431	
.6805		
7.431	Interest on the interest.	
350.00		
$441.82		

PARTIAL PAYMENTS.

Pages 276, 277.

8.

1869	6	10
1870	1	25
	7	15
1870	5	15
	3	20
1871	2	20
	9	5
1871	7	5
	4	15

$504	$346.946
.06	.06
2)30.24	2)20.81676
15.12	10.408
2.52	5.204
1.26	.289
18.90	15.901
504	346.946
522.90	362.847
84.00	200.000
$438.90	$162.847
.06	.06
4)26.3340	3)9.77082
6.583	3.2569
1.463	.407
438.90	3.664
446.946	162.847
100	$166.51
$346.946	

4.

1865	1	1
1865	3	16
	2	15
1866	1	1
	9	15
1866	4	4
	3	3
1867	1	1
	8	27

$450	$306.725	
.08	.08	
6)36.00	4)24.538	
6.00	6.134	
1.50	.204	
450.00	306.725	
457.50	313.063	
75.00	200.00	
$382.50	$113.063	
.08	.08	
2)30.60	2)9.04504	
15.30	4.5225	6 mo.
7.65	1.5075	2 mo.
1.275	.5025	20 da.
382.50	.1256	5 da.
406.725	.0502	2 da.
100.00	6.708	
$306.725	113.063	
	$119.771	

5.

	1870	3	11
	1871	6	5
$12	1	2	24
	1874	12	18
$31	3	6	13
	1876	7	1
$5	1	6	13

$58.50
5.85
.975
.325
.065
58.50
65.715
12
$53.715
5.3715
3½
2.6857 .4476
16.1145
.1492
.0447
53.715
72.71

72.71
31.00
$41.71
4.171
2.0855 .3475
.1158
.0347
41.71
48.117
5
$43.12

6.

$400	$250.96
.07	.07
28	17.5672
400	250.96
428	268.527
100	100.00
$328	$168.527
.07	.07
22.96	3)11.78689
328.00	3.929
350.96	168.527
100.00	$172.46
$250.96	

Pages 277, 278.

2.

8	16	30
30	30	28
31	31	31
30	31	30
31	28	4
31	31	123
28	30	
31	4	
30	201	
4		
254		

$\frac{\overset{7}{\cancel{42}} \times 254}{\overset{}{\cancel{360}}_{60}} = \frac{177.8}{6} =$ $29.64

420.00

$449.64

$\frac{10 \times \overset{67}{\cancel{201}}}{\overset{}{\cancel{360}}_{12}} =$ $5.58

100.00

$105.58

$\frac{\overset{41}{\cancel{20}} \times \cancel{123}}{\overset{}{\cancel{360}}_{\cancel{18}\ 6}} =$ $6.84

200.00

$206.84

105.58

312.42

449.64

$137.22

8. 30
28
31
89
93

$\frac{\$0 \times 93}{\$60} = \$7.75$

500.00

$507.75

Apr. 4, maturity.

11
28
31
4
74

$\frac{74 \times \$}{\$60} = \$1.233$

100.00

$101.233

18
31
4
53

$\frac{53 \times 3}{\$60} = \$0.442$

50.00

$50.442

3
31
4
38

$\frac{38 \times \$}{\$60} = \$0.633$

100.00

$100.633

30
4
34

$\frac{34 \times 9}{\$60} = \$0.85$

150.00

$150.85

$101.233
50.442
100.633
150.85
$403.16
507.75
$104.59

4.

$100	$200	$400
.07	.07	28
2)7.00	2)14.00	0.23
3.50	7.00	$428.23
.06	3.50	314.80
100.00	.39	$113.43
$103.56	.35	
211.24	200.00	
$314.80	$211.24	

The note ran 1 yr. 3 da.

The first payment was on interest, 9 mo. 19 da.

Interest for 19 da. is found by taking $\frac{1}{6}$ of interest for 3 mo., and from this deducting $\frac{1}{10}$ to obtain interest for 9 da. All is done mentally, except what is on the paper.

5.	$200	$350	$700
	.06	.06	.06
	12)12.00	4)21.00	2)42.00
	1.00	5.25	21.00
	0.67	0.58	0.35
	0.20	350.00	700.00
	200.00	$355.83	$721.35
	$201.87	201.87	557.70
		$557.70	$163.65

The note ran 6 mo. 3 da.

The 1st payment was on int., 3 mo. 10 da. 10 da. $= \frac{1}{3}$ of 3 mo.

The 2d payment was on interest, 1 mo. 26 da. 26 da. $= \frac{2}{3} + \frac{1}{5}$ of 1 mo.

Page 280.

2. Interest due at the "rest," July 27, 1874,	$80.00
Amt. of $50 payt. at " " "	52.29
Deferred interest	$27.71
Int. on deferred int. to July 27, 1876 .	4.43
Amt. of $80 int. deferred 1 yr. . . .	86.40
Int. falling due July 27, 1876	80.00
Total int. due July 27, 1876 . . .	$198.54
	1000.00
Amt. due July 27, 1876	$1198.54
Amt. of the $150 payt. + the $200, up to the "rest," July 27, 1876 . . .	360.267
Bal. due July 27, 1876	$838.273

Bal. due July 27, 1876	$838.273
Int. to July 27, 1877	67.062
Amt. due July 27, 1877	$905.335
Amt. of the $250 payt., July 27, 1877 .	259.667
Bal. due July 27, 1877	$645.67
Int. to Sept. 1, 1877	4.88
Bal. due Sept. 1, 1877	$650.55

8. Solution when indorsements are made without the specification "on interest accruing:"—

Int. on $500, 1 yr., 6%, $30, deferred 1 yr., since the $15 paid Feb. 10, 1874, was less than the interest then due.

Amt. of $30 for 1 yr.	$31.80
Amt. of $500 for 1 yr.	530.00
	$561.80

Inasmuch as the sum of the two payments ($15 Feb. 10, 1874, and $25 Aug. 15, 1874) exceeds the interest due at the latter date, this $40 is on interest the remainder of that year (9 mo. 23 da.), and the $150 paid May 17, 1875, is also to be on interest the remainder of the year (21 da.). These amounts are $41.95 and $150.52, or in all $192.47. Hence we have,

Bal. due June 7, 1875, $561.80 — $192.47 =	$369.33
Int. on this bal. for 1 yr.	22.16
Amt. due June 7, 1876	$391.49
The $20 paid Jan. 13, 1876, being more than the interest then due, we take the amt. for 4 mo. 25 da. to the next rest	20.48
Bal. due June 7, 1876	$371.01
Interest on this for 4 mo. 11 da.	8.10
	$379.11

When payts. are made "on interest accruing:"—

Amt. of $500, 1 yr., $530. As by the stipulation the $15

paid this year was "on interest accruing," applying it, there remains due, June 7, 1874, $515.

Amt. of $515 for 1 yr., $545.90. During this year $175 have been paid, paying up the "interest accruing," and leaving $175 — $45.90 = $129.10, as payt. on the principal made May 17, 1875. The amt. of this at the next rest, June 7, 1875 (21 da.), is $129.55. Applying this, there remains $500 — $129.55 = $370.45, due June 7, 1875.

Amt. of $370.45 for 1 yr., $392.68. Deduct the $20 paid during this year, and $372.68 remains due June 7, 1876.

Amt. of $372.68 from June 7, 1876, to Oct. 18, 1876, $380.82.

NOTE. — The legal provision in New Hampshire for indorsements "on interest accruing" is specially designed to cover cases in which payments are made of *less amount than the interest accruing.* Thus, if I owe Mr. A. $2000 on interest at 10%, and can pay him during the year $150 (at one or several payments), but cannot pay an amount equal to the interest accrued, according to the rule, when the provision "on interest accruing" is not made, I should lie out of the use of my money till such time as I could pay up the entire interest accrued. Now, suppose this should continue 5 yr., and I should thus pay $150 per yr., I should lose the use of all the payments from the time they were made to the end of the 5 yr. Hence the debtor will seek to have his payments "on interest accruing" if he cannot meet the entire accrued interest.

If, however, by oversight or otherwise, the debtor has made a payment greater than the interest accrued, and has allowed it to be indorsed as "on interest accruing," only so much of it could be claimed "on interest accruing" as would cover the interest due at the close of that period or year. The next year's interest is not yet accruing. Hence the balance of any such payment would be treated in the ordinary way provided for cases in which payments exceed the interest.

EXAMPLE.

$2000.

Three years after date, for value received, I promise to pay C. N. Jones, or bearer, *two thousand dollars*, with interest payable annually at 10 per cent.

ANN ARBOR, May 7, 1876. W. W. BEMAN.

Indorsements: $150 on the 1st day of April each year.

What was the amount due on this note if the indorsements were "on interest accruing?" What, if this phrase were not inserted?

If "on interest accruing," at May 7, 1874, there would be an unpaid balance of interest of $50. May 7, 1875, there would be $200 + $50 + $5 = $255 interest due. Deducting the $150 would leave $105 interest unpaid. May 7, 1876, there would be $200 + $105 + $10.50 = $315.50 interest due. Deducting the payment $150, and adding the principal, we have $315.50 — $150 + $2000 = $2165.50, the amount due.

If the payments were not specified to be "on interest accruing," the amount due May 7, 1876, would be $200 + $40 + $200 + $20 + $200 + $2000 — $450 = $2210.

TRADE DISCOUNT.

Pages 283, 284.

3. This, in the language of trade, means a deduction of 10% from the face of the bill, and hence is $\frac{1}{10}$ of $250, or $25, and leaves $250 — $25 = $225.

4. 1.00 × .10 = .90. $6.75 ÷ .90 = $7.50. Hence the retail price was $7.50; and $750 ÷ $1.25 = $6.00, was the cost. The reasoning for the first operation is the same as in Ex. 1; for the second it is, each $1.00 cost was marked to retail at 25% advance, i.e., at $1.25. Hence $7.50 retail represented $7.50 ÷ 1.25 of cost.

5. $500 — 6% of $500, or $500 — $30 = $470. $500 — 5% of $500, or $500 — $25 = $475.

6. 1.00 — .20 = .80. $2.20 ÷ .80 = $2.75. As a discount of 20% was allowed, for each $1.00 of the asking price the seller took $1.00 — $0.20 = $0.80. And as I

paid \$2.20, and each \$0.80 of this represented \$1.00 of the asking price, the asking price was \$2.20 ÷ .80 = \$2.75.

7. 5% of \$600 is \$30
4% of \$600 is \$24
\$54

8. On the \$350 I get $3\frac{1}{2}\%$ for anticipating payment 20 da. The question then is, $3\frac{1}{2}\%$ for 20 da. is how much for 1 yr., or $18\frac{1}{4}$ times 20 da.? $18\frac{1}{4}$ times $3\frac{1}{2} = 63\frac{7}{8}$.

At 360 da. to a year, it would be $\frac{360}{20}$, = 18, times $3\frac{1}{2}\%$, or 63%.

For paying \$500, 40 da. before due, I get 5%. Hence for 1 yr. I should get $\frac{365}{40}$, = $9\frac{1}{8}$, times 5%, or $45\frac{5}{8}\%$.

At 360 da. to a year it would be $\frac{360}{40}$, = 9, times 5%, or 45%.

9. \$3.75 — 10% of \$3.75 = \$3.75 — \$0.37½ = \$3.37½.

10. \$3.375 ÷ .90 = \$3.75.

11. $\frac{\$3.75 - \$3.375}{\$3.75} = .10$, or 10%.

12. 15% of \$2157.25 = \$323.59. \$2157.25 — \$323.59 = \$1833.66. \$1833.66 less 3% = \$1778.65.

BANK DISCOUNT.

Pages 285-288.

8. OPERATION.

$$\frac{63 \times \overset{100}{\cancel{\$00}}}{\underset{73}{\cancel{\$0\$}}} = \frac{6300}{73} = 86.30+.$$

Or,

$$\frac{\overset{7}{\cancel{\$3}} \times 50\cancel{\$}}{\underset{4}{\cancel{\$00}}} = \frac{350}{4} = 87.50.$$

EXPLANATION. — The interest for 1 yr. is $500. For 63 da., calling 365 da. a year, $\frac{63 \times 500}{365}$; calling 360 da. a year, $\frac{63 \times 500}{360}$.

NOTE. — There can be no question but the above forms are most expeditious for such problems, and pupils should be made familiar with them.

4. $\frac{\overset{1.75}{\not{1}\not{7}\not{5}} \times \not{1}\not{0} \times \overset{7}{\not{6}\not{3}}}{\underset{4}{\not{3}\not{6}\not{0}} \times \not{1}\not{0}\not{0}} = \frac{12.25}{4} = \$3.06.$

$175 — $3.06 = $171.94.

5. The bank would compute the amt. of the note Dec. 13, 1875 (i.e., for 1 yr. 10 mo. 18 da.), at 7%. This is $565.92. This would then be discounted at 10% for the exact number of days between Sept. 6, 1875, and Dec. 13, 1875; i.e., for 98 da. The discount is $15.406. Hence the avails are $565.92 — $15.46 = 550.46.

6. (1) Is due June 9, 1879. The amt. of $350 at 7% for 4 mo. 3 da. is $358.37. This, discounted at 8% from April 1 to June 9, 69 da., gives as avails $352.88.

(2) Is due May 4, 1879. The amt. of $300 at 6% for 2 mo. 3 da. is $303.15. This, discounted at 8% from April 1 to May 4, 33 da., gives as avails $300.93.

(3) Is due June 15, 1879. The amt. of $150 for 3 mo. 3 da. at 8% is $153.10. This, discounted at 8% from April 1 to June 15, 75 da., gives as avails $150.55.

Hence the total avails is $352.88 + $300.93 + $150.55 = $804.36.

7. The note is due June 13, 1879, and hence is discounted for 95 da., or from March 10 to June 13.

$\frac{\$320 \times 7 \times 95}{360 \times 100} = \$5.91$, the discount. $320 — $5.91 = $314.09, the avails.

8. The amount of the note, which falls due March 19, 1879, is \$784.125. This sum, discounted for 146 da., gives a discount of \$19.08. The work is $\frac{\$750 \times 6 \times 9}{100 \times 12} + \frac{\$750 \times 6 \times 3}{100 \times 360}$ = \$34.125, the interest. \$750 + \$34.125 = \$784.125, the amt. of note. This is discounted thus:

$$\frac{\$784.125 \times 6 \times 146}{100 \times 360} = \$19.08.$$

Nos. **9, 10, 11, 12,** and **13** are solved in the same way. We give the figures as they would appear in working the 13th:

\$6000
.09
2)540.00
60)270.00 Int. 6 mo.
4.50 " 3 da., $\frac{1}{60}$ of 6 mo.
274.50
6000
\$6274.50 Amt. of note when due,
.07 i.e., Nov. 13, 1868.

360)439.2150(1.22
36 41
79 122
72 488
72 50.02
72 6274.50
\$6224.48

14. $\frac{63 \times 7 \times \overset{50}{250}}{\underset{73}{365} \times \underset{2}{100}} = \frac{441}{146} = 3.02.$

$\frac{33 \times 7 \times \overset{5}{250}}{\underset{73}{365} \times \underset{2}{100}} = \frac{231}{146} = 1.58+.$

$\frac{93 \times \overset{5}{25}}{\underset{73}{365}} = \frac{465}{73} = 6.37+.$

$\frac{83 \times \overset{5}{25}}{\underset{73}{365}} = \frac{165}{73} = 2.26+.$

Pages 290, 291.

2. Int. of \$1 for 33 da. at 8% is $\frac{\overset{.02}{.08} \times \overset{11}{33}}{\underset{\substack{126 \\ 30}}{360}} = \frac{.22}{30} = .007\frac{1}{3}$.

\$1 − \$0.007⅓ = \$0.992⅔. \$50 ÷ .992⅔ = \$50.37−.

8. $\frac{.07 \times \overset{31}{93}}{\underset{120}{360}} = \frac{.217}{12} = .018\frac{1}{12}$. 1 − .018$\frac{1}{12}$ = .981$\frac{11}{12}$.

\$1000 ÷ .981$\frac{11}{12}$ = \$1018.41+.

4. $\frac{\overset{.01}{.09} \times \overset{12}{48}}{\underset{\substack{40 \\ 10}}{360}} = .012$. 1 − .012 = .988. \$2500 ÷ .988 = \$2530.36. \$2530.36 − \$2500 = \$30.36+.

7. \$250 + \$175 + \$120 = 545. $\frac{\overset{.01}{.06} \times \overset{31}{93}}{\underset{\substack{125 \\ 20}}{360}} = \frac{.31}{20} = .0155$.

1 − .0155 = .9845. \$545 ÷ .9845 = \$553.58+.

9. $\frac{\overset{.02}{.08} \times \overset{7}{63}}{\underset{\substack{40 \\ 10}}{360}} = .014$. 1 − .014 = .986. \$2800 ÷ .986 = \$2839.76−. \$2850 − \$2839.66 = \$10.34.

TRUE DISCOUNT.

Pages 293, 294.

4. The amount of the note when due (i.e., Dec. 10, 1879) is \$314.778. This, discounted for 6 mo. 27 da. (i.e., from May 13, 1879, to Dec. 10, 1879), gives \$314.778 ÷ 1.046 = \$300.93+.

The work is as follows:

1879	12	10	\$276	
1877	8	7	.06	
2	4	3	16.56	
			$2\frac{1}{2}$	
3 da. = $\frac{3}{30}$ = $\frac{1}{10}$ mo.			33.12	
$\frac{1}{10}$ mo. = $\frac{1}{10}$ of $\frac{1}{4}$ = $\frac{1}{40}$ of 4 mo.			5.52	
			.138, $\frac{1}{40}$ of 4 mo. int.	
1879	12	10	38.778	
1879	5	13	276.00	
	6	27	314.778	(1.046
			3138	300.93
$\frac{6}{12}$ of .08 = .04			9780	
$\frac{27}{360}$ of .08 = .006			9414	
	.046		3660	
			3138	
1 + .046 = 1.046			522	

5. The amt. of the note, Sept. 20, 1879, at 7% (i.e., the amt. of \$186.50 at 7% for 3 yr. 7 mo. 13 da.), is \$233.7518. The amt. of \$1 at 10% from July 17, 1878, to Sept. 20, 1879 (i.e., 1 yr. 2 mo. 3 da.), is \$1.1175. \$233.7518 ÷ 1.1175 = \$209.17+.

6. The note is to run 2 yr. 5 mo. 14 da., and, at 6%, amounts to \$401.56$\frac{2}{3}$. \$1 at 10% from Jan. 14, 1875, to Oct. 21, 1876 (i.e., 1 yr. 9 mo. 7 da.), amounts to \$1.1769$\frac{4}{5}$.

Hence, at Jan. 14, 1875, discounting at 10%, the note was worth $401.56⅔ ÷ 1.1769⅓ = $341.194+
But the speculator offered 300.00
That is $41.194+
less than the note was worth.

OPERATION.

1876 10 21 — $350
1874 5 7 — .06
2 5 14 — 21.00
30 — 2
42.00
$\frac{41\ \not{1}\not{0}\not{4} \times \overset{7}{\not{2}\not{1}}}{\not{3}\not{6}\not{0}} = \frac{287}{30} = 9.56\frac{2}{3}$ — 9.56⅔
350.00
1876 10 21 — $401.56⅔
1875 1 14 — 1.1769⅓ =
1 9 7
30 — $1.00 — $3614.10
$\frac{2.77 \times .1\not{0}}{3\not{6}\not{0}} =$.10 — 1059.25 =
.769⅓
$1.1769⅓ — $341.194+.

7. OPERATION.

1879 2 6
1876 9 17
2 4 19 — 28.00
1878 5 23 — 2⅓
8 13 — 56.00
.08 — 9.3333
.04 — .7777
.01333 — .70
.00222 — 66.8111
.00066 — 280.00
1.05622)346.81111(328.35

$$
\begin{array}{r}
1.05622)346.81111(328.35 \\
316.86666 \\
\hline
2994444 \\
2112444 \\
\hline
882000 \\
844977 \\
\hline
370222 \\
316866 \\
\hline
53355 \\
52811 \\
\hline
544
\end{array}
$$

8. Amt. of \$300 for 2 yr. at 10% = \$300 + \$60 = \$360. \$360 ÷ 1.16 = \$310.34+.

9. \$348 = 1.2 = \$290.

10. (1) As the note bears interest for all money is worth, it is worth its face the day it is made; for, if the holder were to sell it for less or more, he would, at 10% on the avails, have less or more than \$300 at the expiration of the 3 yr.

(2) The amt. of the note at the end of the time (3 yr.) is \$390. If it is sold at the end of the first year, the purchaser has to wait 2 yr. for the \$390, and hence this amt. is to be discounted for 2 yr.; \$390 ÷ 1.2 = \$325.

(3) \$390 ÷ 1.05 = \$371.43−.

11. (1) \$300 ÷ 1.3 = \$230.78−.

(2) \$300 ÷ 1.2 = \$250.

(3) \$300 ÷ 1.05 = \$285.71+.

(4) 3 yr. after date it is due, and is worth just \$300.

12. $\frac{\$4200}{1.04} = \$4038.46+$.

13. Amt. of \$52.25, at 10%, for 6 yr. 3 mo. 19 da., \$85.18202. This discounted at 6% for 1 yr. 6 mo., is \$85.18202 ÷ 1.09 = \$78.148+.

14. This, in the language of trade, means a deduction of

8% from the face of the bill, and hence is 8% of $750, or $60, and leaves $750 — $60 = $690.

17. Amt. of 1st note, 1 yr. 4 mo. 24 da., $243.90.

$243.90 discounted at 8% for 8 mo. 21 da. . . .	$230.53
Amt. of 2d note, 2 yr. 6 mo. 2 da., 183.36\frac{1}{15}$.	
183.36\frac{1}{15}$ discounted at 8% for 1 yr. 23 da. . .	168.98
Amt. of 3d note, Apr. 10, 1879, 1 yr. 2 mo. 7 da.,	216.61
	$616.12

GOVERNMENT BONDS.

Pages 298-301.

1. $1.165 × 500 = $582.50.

2. $0.345 × 1000 = $345.

3. 103$\frac{5}{8}$ means 1.03\frac{5}{8}$, or $1.03625, per $1.00 of bond. $1.03625 × 1000 = $1036.25. But as the interest on these bonds is payable Feb. 1, May 1, Aug. 1, and Nov. 1, on May 20 there is 19 da. interest accrued. This at 5% is $\frac{\$1000 \times 5 \times 19}{365 \times 100} = \$2.60$. $1036.25 — $2.60 = $1033.65.

6. $1.11875 × 100 = $111.875. The accrued interest on a $100 bond is $\frac{1}{4}$ of $7 = $1.75. $111.875 + $1.75 = $113.625.

7. $0.86875 × 1000 = $868.75. The accrued interest on a $1000 bond is $\frac{2\frac{1}{2}}{12}$, or $\frac{5}{24}$, of $36.50 = $7.60. $868.75 + $7.60 = $876.35.

10.

First cost, 1.02\frac{1}{4}$ × 2500 =	$2556.25
$\frac{1}{8}$% of $2500 brokerage	5.00
Total amount paid	$2561.25
54 da. int. of $2500 at 4%	14.79
Actual cost above accrued interest .	$2546.46

12. $\$1.03625 \times 1000 =$ $\$1036.25$

Brokerage, $\frac{1}{10}\%$ of $\$1000 =$ 1.00

Paid for the bond $\$1037.25$

Int. for 73 da. at 5% on $\$1000$ 10.00

Net cost $\$1027.25$

16. An investment of $\$103\frac{1}{4}$ buys a $\$100$ bond which yields $\$4$ per annum. Hence the rate is $4 \div 103\frac{1}{4} = .0387+$, or $3.87\%+$.

17. The accrued interest on $\$100$ of the U. S. bonds is 75 cents. Hence the actual cost is $\$104.50 - \$0.75 = \$103.75$. This makes the U. S. investment yield $\frac{450}{103.75} = 4.337\%+$, or $4\frac{1}{3}+\%$.

On the State 7s the accrued int. is $\frac{5}{12}$ of $\$7.00$, or $\$2.91\frac{2}{3}$ on $\$100$. This makes the cost of the bond $\$107 + \$2.91\frac{2}{3} = \$109.91\frac{2}{3}$. Hence these yield $700 \div 109.91\frac{2}{3} = 6\frac{1}{3}\%+$.

DOMESTIC EXCHANGE.

Pages 303-305.

2. $\$1500 \times \frac{1}{12}\%$ of $\$1500 = \$1501.25$.

3. $\$1825 - \frac{1}{20}\%$ of $\$1825 = \$1824.09$.

4. $\frac{\$2500}{1.001} = \$2497.502+$.

6. $\frac{\$1500}{1.00\frac{1}{8}} = \frac{\$12000}{8.01} = \$1498.13+$.

7. $\$3000 + \frac{1}{8}\%$ of $\$3000 = \$3003.75$.

(The following are computed at 365 da. to a year, as this is the method of New-York City banks.)

10. $\$1.00 + .02 = \$1.02$; $\$0.05 \times \frac{63}{365} = \frac{63}{73}¢$, the time discount.

$\$1.02 - .00\frac{63}{73} = \$1.01\frac{10}{73}$, cost of $\$1$ of exchange.

$\$1.01\frac{10}{73} + 750 = \$758.53$, cost of $\$750$ exchange, besides $2¢$ stamp.

$\$1.01\frac{10}{73} \times 85.50 = \$86.47$, cost of \$85.50 exchange, besides stamp.

11. Time discount on \$1 is $4¢ \times \frac{93}{365}$, or $1\frac{7}{365}¢$. To this adding the exchange, we have total discount on \$1, $1\frac{7}{365}¢ + 1\frac{1}{2}¢ = 2\frac{378}{730}$. Hence \$1 exchange costs $\$1.00 - \$0.02\frac{378}{730}$, or $\$0.97\frac{351}{730}$.

Cost of \$500, 500 times $\$0.97\frac{351}{730}$, or \$487.40, without stamp.

Cost of \$184.25, 184.25 times $\$0.97\frac{351}{730}$, or \$179.63, including $2¢$ for stamp.

12. Time discount, $\frac{33}{365}$ of $5¢$, or $\$0.00\frac{33}{73}$. Hence,

$\$1.01\frac{3}{4} - \$0.00\frac{33}{73} = \$1.01\frac{87}{292}$, or $100\frac{87}{292}\%$.
$\$1.01\frac{3}{4} - \$0.00\frac{83}{73} = \$1.00\frac{259}{292}$, or $100\frac{259}{292}\%$.
$\$1.01\frac{3}{4} - \$0.00\frac{93}{73} = \$1.00\frac{139}{292}$, or $100\frac{139}{292}\%$.
$\$1.01\frac{3}{4} - \$0.00\frac{13}{73} = \$1.01\frac{167}{292}$, or $100\frac{167}{292}\%$.

13. $4.5 \times \frac{33}{365} = \frac{297}{730}$; $98.5 - \frac{297}{730} = 98\frac{34}{365}$.
$4.5 \times \frac{63}{365} = \frac{567}{730}$; $98.5 - \frac{567}{730} = 97\frac{264}{365}$.
$4.5 \times \frac{93}{365} = 1\frac{107}{730}$; $98.5 - 1\frac{107}{730} = 97\frac{120}{365}$.
$4.5 \times \frac{13}{730} = \frac{117}{730}$; $98.5 - \frac{117}{730} = 98\frac{124}{365}$.

FOREIGN EXCHANGE.

Page 306.

2. As my bill will probably reach London in time to allow 60 da. after presentation before Aug. 1, a 60 da. bill of exchange will answer my purpose in the first case. This will cost me $\$4.87\frac{1}{2} \times 325 = \$1584.37\frac{1}{2}$.

In the second case I shall need a "sight" draft, or bill. This will cost me $\$4.89 \times 325 = \$1589.25$.

3. $5^{fr}17875 \times 500 = 2579^{fr}375$.

4. $\$0.95\frac{3}{4} \times 250 = \$239.37\frac{1}{2}$.

GENERAL PROBLEMS IN PERCENTAGE.

Pages 308, 309.

7. The *effect* is \$102.81 — \$74.50 = \$28.31. The thing required is *rate per cent.* \$74.50 at 1% for 3 yr. 9 mo. 18 da. = $3 + \frac{9}{12} + \frac{18}{360}$ (or $\frac{1}{20}$) yr. = $3\frac{1}{5}$ yr. is \$0.745 × $3\frac{1}{5}$ = \$2.831. \$28.31 ÷ \$2.831 = 10.

8. \$75 at 1% for $6\frac{1}{4}$ yr. is \$4.6875. \$28.125 ÷ \$4.6875 = 6. \$15.30 ÷ \$1.50 = $10\frac{1}{5}$.

9. \$942 — \$750 = \$192. \$750 at 6%, 1 yr., gives as interest \$45. \$192 ÷ \$45 = $4\frac{4}{15}$, or 4 yr. 3 mo. 6 da.

10. \$1 yields $28\frac{5}{6}¢$ in 7 yr. 2 mo. 15 da. at 4%. Hence \$150 ÷ $.28\frac{5}{6}$ = \$520.23 is the principal required.

11. In 1 yr. \$120 yields \$7.20 at 6%. Hence the required time, 16.56 ÷ 7.20, or 2 yr. 3 mo. 18 da.

14. Int. of \$1 at 8% for 33 da. = $.007\frac{1}{3}$. \$1 = \$0.$007\frac{1}{3}$ = \$0.$998\frac{2}{3}$. \$350 ÷ $998\frac{2}{3}$ = \$352.58+.

15. $\frac{10¢ \times 33}{360} = .9\frac{1}{6}¢$. \$1.00 — \$0.$009\frac{1}{6}$ = \$0.$990\frac{5}{6}$.

\$495.50 ÷ $.990\frac{5}{6}$ = \$500.08+.

16. The discount was \$125 — \$122.90 = \$2.10. At 1% per annum the discount on \$125 for $^{30}/_{33}$ da. is \$1.25 × $\frac{33}{360}$ = \$0.$11458\frac{1}{3}$. Hence the rate per cent was 2.10 ÷ $.11458\frac{1}{3}$ = $18\frac{1}{3}\%-$.

17. \$1500 ÷ .07 = \$21428.57+.

18. \$500 ÷ .01 = \$50000.

19. \$400 ÷ $\frac{.05}{4}$ = \$32000.

20. \$600 ÷ .01 = \$60000.

RULE OF THREE.

Pages 315-317.

Note. — In solving problems by proportion, it is essential that the pupil see and give the reason for the equality of the ratios. The old mechanical method of "stating" the proportion by "seeking for the third term that which is of the same name as the answer sought, and then considering whether the answer is to be greater or less than this, and, if it is to be greater, making the greater of the two remaining terms the second term, and the other the first," etc., is almost sure to lead to entire neglect of the principles at the bottom of the application of the proportion. Observe the notes and questions in the text-book on pp. 314, 315, and the following explanations.

4. Now it is evident, that, if the land all produces alike, there will be the same ratio between the number of bushels produced on two fields of wheat as there is between the size of the fields: i.e., if one field is twice as large as the other, it will produce twice as much; if three times as large, it will produce three times as much. Now, in this example, the ratio of the first field to the second (the one inquired about) is 12 : 36; then there must be the same ratio between the number of bushels produced on the first field to the number produced on the second, i.e., the ratio 384 bu. : the No. bu. produced on the 2d field. Hence the ratio 12 : 36 equals the ratio 384 bu. : No. bu. on 2d field, or

12 A. : 36 A. : 384 bu. : No. bu. on 2d field.

To solve this proportion so as to find the 3d term, we may proceed in either of three ways:

1st. As the consequent of the 1st ratio, 12 : 36, is 3 times the antecedent, the consequent of the 2d ratio must be 3 times its antecedent. Hence the 4th term is 3 times 384, or 1152, and 36 A. produce 1152 bu.

2d. As the product of the extremes is equal to the product of the means, we observe that the product of the means is 36×384, or 13824. Then we know the 1st term, 12, and

the product of this and the 4th. Now, knowing the product, 13824, and one of the factors, 12, to find the other factor, we divide the product by the given factor. Hence the 4th term is $\frac{13824}{12} = 1152$.

3d. As in the last, we consider that the product of the means is 36×384, and that this divided by the given extreme, 12, gives the other extreme, and so have the 4th term equal to $\frac{\overset{3}{\cancel{36}} \times 384}{\cancel{12}} = 1152$. This is the same as the 2d in the reasoning, and the same as the 1st in the work.

REMARK. — The first method should be used till it is perfectly familiar, as it exhibits best the nature of a proportion. The third is the most convenient for practice.

$$\overset{\text{Prin.}}{\$350} : \overset{\text{Prin.}}{\$700} :: \overset{\text{Int.}}{\$65} : \text{——}.$$

As the 1st consequent is 2 times the 1st antecedent, the 2d consequent must be 2 times the 2d antecedent, or $2 \times \$65 = \$130$.

Or, $$\frac{\overset{2}{\cancel{700}} \times \$65}{\cancel{\$50}} = \$130.$$

8. \$100 now amounts to $\$110.79\frac{1}{6}$, 1 yr. 6 mo. 15 da.; hence at 7%. Hence the question is, If \$100 principal amounts to $\$110.79\frac{1}{6}$ in a given time at a given rate, what principal will amount to \$350 in the same time at the same rate? The statement is, —

$$\overset{\text{Amt.}}{\$110.79\tfrac{1}{6}} : \overset{\text{Amt.}}{\$350} :: \overset{\text{Prin.}}{\$100} : \overset{\text{Prin.}}{\text{——}}.$$

$$\therefore \frac{350 \times \$100}{110.79\frac{1}{6}} = \frac{\$210000}{66475} = \$315.91-.$$

9. $$\overset{\text{Amt.}}{\$148} : \overset{\text{Amt.}}{\$725\tfrac{1}{2}} :: \overset{\text{P.W.}}{100} : \overset{\text{P.W.}}{\text{——}}.$$

$$\therefore \frac{725.5 \times \overset{25}{\cancel{\$100}}}{\underset{37}{\cancel{148}}} = \frac{\$18137.5}{37} = \$490.20+.$$

10. Amt. Amt. P.W. P.W.
$124.50 : $2000 :: $100 : ——.

$$\therefore \frac{\$200000}{124.5} = \$1606.43.$$

11. $1 : $569.31 :: $0.43 : ——.

12. Difference in longitude, 5° 57′ 10″.

$$\therefore\ 15° : 5°\ 7'\ 10'' :: 1 \text{ hr.} : ——.$$

Or,

$$900' : 357\tfrac{1}{6}' :: 1 \text{ hr.} : ——.$$

$$\therefore \frac{3.57\frac{1}{6}}{9} \text{ hr.} = \frac{3.57\frac{1}{6} \times 60}{9} \text{ min.} = \frac{214.3}{9} \text{ min.}$$

13. In. In. Yd. Yd. In.
36 : (36 − $\frac{1}{8}$) :: 500 : 498 $9\frac{1}{2}$, Ans.

14. $13 : 31 :: 10 : 23\frac{11}{13}$, Ans.

15. The number of days required is in inverse ratio to the length of the days; i.e., the longer the days, the less number will be required. Hence,

Hr. Hr. Da.
$11\frac{1}{4} : 10 :: 13\frac{1}{2} : ——.$

$$\therefore \frac{13\frac{1}{2} \times 10}{11\frac{1}{4}} = \frac{\overset{6}{\cancel{54}} \times \overset{2}{\cancel{10}}}{\underset{9}{\cancel{45}}} = 12.$$

16. 12 in. : 39.37 in. :: 332.05 m. : ——. That is, as 1 ft. (12 in.) is to 1 meter (39.37 in.), so is the number of meters to the number of feet. The number of meters is to the number of feet in inverse ratio of the lengths of a meter and a foot; the longer the measure (foot or meter), the less the number in any given distance.

Remark. — No one can fail to observe that most of the problems usually solved by proportion (or the "Rule of Three") in arithmetic are more readily solved, without the idea of a proportion, by what is called "Analysis." The truth is, for such solutions the proportion is a clumsy machine: nevertheless, the proportion is an important instrument in higher mathematics, and its nature and properties should be learned.

17. If sound travels 1089.4 ft. per second, how far does it travel in 15 sec.?

1 sec. : 15 sec. :: 1089.4 : ——.

15 × 1089.4 ft., or 3.1 mi. nearly.

18. If the passage of light was instantaneous, one of Jupiter's moons would disappear from our sight exactly at the instant that it passed behind the disk of the planet; but, if the passage of light is not instantaneous, the moon will be seen by us *after* it has passed behind the planet's disk till all the light which started from the moon before it passed behind the disk has reached us. Now this will be a longer or shorter time according as we are nearer to or farther from the planet. Hence we learn from the fact stated in the example that it takes light 16 min. 26.6 sec. to cross the earth's orbit, a distance of 183000000 miles. Thus the question is, If light travels 183000000 miles in 16 min. 26.6 sec., how far does it travel in 1 sec.?

16 min. 26.6 sec. : 1 sec. :: 183000000 mi. : ——.

Or, 986.6 sec. : 1 sec. :: 183000000 mi. : ——.

$$\therefore \frac{183000000 \text{ mi.}}{986.6} = 185485 \text{ mi.} + \text{ per second.}$$

19. 24000 mi. : 185485 mi. :: 1 : ——.

The number of times it will go round in a given time is in the inverse ratio to the distance around. Thus, if light would go around a globe 185485 mi. in circumference in 1 sec., how many times will it go around a globe 24000 mi. in 1 sec.?

$$\therefore \frac{185485}{24000} = 7\tfrac{3}{4} \text{ nearly.}$$

20. 185,485 : 100,000,000,000,000 :: 1 sec. : ——.

$$\frac{100,000,000,000,000}{185,485} \text{ sec.} = 17 \text{ yr. } 34 \text{ da. } 21 \text{ hr. } 32 \text{ min. } 33 \text{ sec.}$$

21. The height of the steeple and of the staff are to each other as the lengths of their shadows.

$$\therefore\ 3 \text{ ft.} : 90 \text{ ft.} :: 5 \text{ ft.} : \text{——}.$$

And 30×5 ft. $= 150$ ft. is the height of the steeple.

PARTITIVE PROPORTION.

Pages 317, 318.

2. $7 : 3 :: 35 :$ 1st part.
$7 : 4 :: 35 :$ 2d part.

$7 : 2 :: 35 :$ 1st part.
$7 : 5 :: 35 :$ 2d part.

3. $(\frac{1}{2} + \frac{2}{3} + \frac{1}{5}) : \frac{1}{2} :: 1 :$ 1st part;
or, $\frac{15 + 20 + 6}{30} : \frac{15}{30} :: 1 :$ 1st part.

Or, $41 : 15 :: 1 : \frac{15}{41}$, 1st part.
Thus $41 : 20 :: 1 : \frac{20}{41}$, 2d part;
and $41 : 6 :: 1 : \frac{6}{41}$, 3d part.

7. For each \$1 A has, B has \$2, C \$4, D \$5, and E \$7.

$\therefore$ 19 : 1 :: \$1900 : \$100, A's share;
19 : 2 :: \$1900 : \$200, B's share, etc.

8. $(\frac{2}{3} + \frac{3}{4}) : \frac{2}{3} ::$ \$85 : A's part;
or, 17 : 8 :: \$85 : \$40, etc.

9. $(340 + 460 + 500) : 340 ::$ \$390 : A's share, etc.

11. $(120 + 160 + 180 + 140) : 120 ::$ \$120 : A's part;
or, 600 : 120 :: \$120 : \$24, A's part;
600 : 160 :: \$120 : \$32, B's part, etc.

13. As often as the 1st has \$3, the 2d is to have \$2, and the 3d is to have $\frac{2}{3}$ of \$2, or \$$2\frac{2}{3}$. Hence,

$7\frac{2}{3} : 3 ::$ \$1000 : 1st part;
or, 37 : 15 :: \$1000 : \$$405\frac{15}{37}$, 1st part, etc.

COMPOUND PROPORTION.

Page 320.

For the method ordinarily known as Compound Proportion there is absolutely no use in mathematics: it is far better every way that problems ordinarily solved in this way should be analyzed in detail. Nevertheless, if any one will use the process of compounding, *it should be shown clearly why the ratios are compounded.* It is solely for the presentation of this thought that the subject is allowed any place. The ordinary mechanical way of treating this subject is worse than useless: it is positively vicious in an educational course. We shall explain in detail Ex. **12**, and give the statement of others.

	Hr.	Da.	Length.	Width.	Depth.	Men.
Given case,	8	18	95	12	9	24
Required case,	12	24	380	9	6	x

Let us take the conditions in their order:

1st. If 24 men are required, working 8 hr. per day, how many will be required when they work 12 hr.? The single point we are to consider is, whether the number of men required will be in the ratio of the number of hours worked per day, or in the inverse ratio. Evidently in the inverse ratio; i.e., the more hours they work per day, the less men it will take. Hence, *if this were the only condition*, we should have $\frac{12}{8} = \frac{24}{x}$.

2d. But the required men are to work more days. Now will this make the ratio of the given number of men to the required number $\left(\text{i.e., } \frac{24}{x}\right)$ greater or less? Evidently it will make the number of men required *less;* and, as it makes x less, it makes the ratio $\frac{24}{x}$ *greater*. But how much? If there were to be *twice* as many days, it would make the number of men

only $\frac{1}{2}$ as great, and hence the ratio $\frac{24}{x}$ twice as great. As it is, there are to be $\frac{24}{18}$ times as many days; whence it will make the ratio $\frac{24}{18}$ times as great as it was before, and we have

$$\frac{24}{18} \text{ times } \frac{12}{8} = \frac{24}{x}$$

when this consideration is involved.

3d. Again, we can in like manner show, that, the length being increased (from 95 to 380), $\frac{380}{95}$ *more men* will be required (i.e., x will be made greater); and hence the ratio $\frac{24}{x}$ will be diminished (it will be only $\frac{95}{380}$ of what it would have been without this condition), and we have

$$\frac{95}{380} \text{ times } \frac{24}{18} \text{ times } \frac{12}{8} = \frac{24}{x}.$$

4th. In a similar manner we show, that, the width being diminished (from 12 to 9 ft.), only $\frac{9}{12}$ as many men would be required (i.e., x would be made only $\frac{9}{12}$ as great). Hence the ratio $\frac{24}{x}$ is made $\frac{12}{9}$ as great, and we have

$$\frac{12}{9} \text{ times } \frac{95}{380} \text{ times } \frac{24}{18} \text{ times } \frac{12}{8} = \frac{24}{x}.$$

5th. In exactly the same manner we see that the depth, being only $\frac{6}{9}$ as great, will require only $\frac{6}{9}$ as many men, or the ratio $\frac{24}{x}$ will be made $\frac{9}{6}$ as great; and we have

$$\frac{9}{6} \text{ times } \frac{95}{380} \text{ times } \frac{12}{9} \text{ times } \frac{24}{18} \text{ times } \frac{12}{8} = \frac{24}{x};$$

or,

$$\frac{\cancel{9}^{3}}{\cancel{6}_{2}} \times \frac{\cancel{95}^{1}}{\cancel{380}_{4}} \times \frac{\cancel{12}^{4}}{\cancel{9}} \times \frac{\cancel{24}^{2}}{\cancel{18}_{3}} \times \frac{\cancel{12}^{4}}{\cancel{8}} = \frac{24}{x}, \text{ or } 1 = \frac{24}{x}, \; x = 24.$$

2. $\left.\begin{matrix}150 : 275\\ 10 : 12\end{matrix}\right\} :: 5.$ $\dfrac{275 \times 12 \times 5}{150 \times 10} = 11.$

3. $\left.\begin{matrix}5 : 20\\ 9 : 30\frac{1}{2}\end{matrix}\right\} :: 1.$ $\dfrac{20 \times 30\frac{1}{2}}{5 \times 9} = 13\frac{5}{9}.$

4. $\left.\begin{matrix}4 : 16\\ 2 : 7\end{matrix}\right\} :: 64.$ $\dfrac{7 \times 16 \times 64}{2 \times 7} = 896.$

5. $\left.\begin{matrix}3 : 33\\ 13 : 14\frac{1}{4}\end{matrix}\right\} :: 100.$ $\dfrac{33 \times 14\frac{1}{4} \times 100}{3 \times 13} = 1205\frac{10}{13}.$

6. $\left.\begin{matrix}2 : 13\\ 1\frac{1}{2} : 1\frac{3}{4}\end{matrix}\right\} :: 10.25.$ $\dfrac{13 \times 1\frac{3}{4} \times 10\frac{1}{4}}{2 \times 1\frac{1}{2}} = \dfrac{13 \times 7 \times 41}{48} =$ 77.73—.

7. $\left.\begin{matrix}13 : 10\\ 200 : 500\end{matrix}\right\} :: 2 : 3\frac{11}{13},$ Ans.

8. $\left.\begin{matrix}6000 : 12000\\ 300 : 100\end{matrix}\right\} :: 2 : 1\frac{1}{3},$ Ans.

9. $\left.\begin{matrix}20 : 7\\ 84\frac{4}{5} : 245\frac{2}{5}\\ 10 : 8\end{matrix}\right\} :: 12\frac{1}{2} : 10\frac{1}{2},$ Ans.

10. $\left.\begin{matrix}18 : 8\\ 12 : 9\\ 120 : 162\\ 9 : 12\\ 3 : 9\end{matrix}\right\} :: 40 : 72,$ Ans.

11. $\left.\begin{matrix}100 : 240\\ 6 : 8\\ 27 : 81\\ 24 : 40\\ 27 : 36\\ 12 : 18\end{matrix}\right\} :: 18 : 256,$ Ans.

12. $\dfrac{24 \times 8 \times 18 \times 380 \times 9 \times 6}{12 \times 24 \times 95 \times 12 \times 9} = 24,$ Ans.

INVOLUTION.

Page 322.

8. $(23)^2 = 23 \times 23 = 529.$

$(341)^2 = 341 \times 341 = 116281.$

$(3580)^2 = 3580 \times 3580 = 12816400.$

$(2\frac{3}{4})^2 = 2\frac{3}{4} \times 2\frac{3}{4} = \frac{11}{4} \times \frac{11}{4} = \frac{121}{16} = 7\frac{9}{16};$

or, $2\frac{3}{4}$

$2\frac{3}{4}$

$\frac{9}{16} + \frac{9}{16}$

$4\frac{6}{4}$

$7\frac{9}{16}$

$\frac{3}{4}$ of $\frac{3}{4}$ is $\frac{9}{16}$, $\frac{3}{4}$ of 2 is $\frac{6}{4}$, 2 times $\frac{3}{4}$ is $\frac{6}{4}$, 2 times 2 is 4. Adding these partial products, we have $7\frac{9}{16}$.

$(4.3)^2 = 4.3 \times 4.3 = 18.49.$

$(\frac{6}{7})^2 = \frac{6}{7} \times \frac{6}{7} = \frac{36}{49}.$

$(.35)^2 = .35 \times .35 = .1225.$

EVOLUTION.

Page 323.

8.

5)11025
5)2205
3)441
3)147
7)49
7

$\therefore$ $11025 = (5 \times 3 \times 7)\ (5 \times 3 \times 7)$, and $\sqrt{11025} = 5 \times 3 \times 7 = 105.$

10)32400
10)3240
2)324
2)162
9)81
9

$\therefore$ $32400 = (10 \times 2 \times 9)\ (10 \times 2 \times 9)$, and $\sqrt{32400} = 10 \times 2 \times 9 = 180.$ When we observe two equal factors, we can remove them whether they are prime factors or not.

5.
2)46656
2)23328
2)11664
2)5832
2)2916
2)1458
3)729
3)243
3)81
3)27
3)9
3

$\therefore\ 46656 = (2 \times 2 \times 3 \times 3) \times (2 \times 2 \times 3 \times 3) \times (2 \times 2 \times 3 \times 3)$, and $\sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36$.

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11.
$\dot{6}7\dot{1}8\dot{4}6\dot{4}(2592$
4
45)271
225
509)4684
4581
5182)10364
10364

12.
$9\dot{6}6\dot{2}8\dot{9}(982.94+$
81
181)1562
1504
1962)5889
3924
19648)186500
176841
196569)965900
786336
179564

16.
$.\dot{0}2\dot{5}6(.16$
1
26)156
156

17.
$\dot{5}.\dot{3}2(2.806+$
4
48)132
129
4606)30000
27636
2364

18.
$$\begin{array}{r} 28.60(5.347+ \\ 25 \\ \hline 103)360 \\ 309 \\ \hline 1064)5100 \\ 4256 \\ \hline 10687)84400 \\ 74809 \\ \hline 9591 \end{array}$$

19.
$$\begin{array}{r} 34.30(5.85+ \\ 25 \\ \hline 108)930 \\ 864 \\ \hline 1165)6600 \\ 5825 \\ \hline 775 \end{array}$$

20.
$$\begin{array}{r} 7.50(2.73+ \\ 4 \\ \hline 47)350 \\ 329 \\ \hline 543)2100 \\ 1629 \\ \hline 471 \end{array}$$

21.
$$\begin{array}{r} 4.90(2.21+ \\ 4 \\ \hline 42)90 \\ 84 \\ \hline 441)600 \\ 441 \\ \hline 59 \end{array}$$

22.
$$\begin{array}{r} .0049(.07 \\ 49 \end{array}$$

23.
$$\begin{array}{r} .0490(.221+ \\ 4 \\ \hline 42)90 \\ 84 \\ \hline 441)600 \\ 441 \\ \hline 59 \end{array}$$

26. $\sqrt{\frac{49}{121}} = \frac{\sqrt{49}}{\sqrt{121}} = \frac{7}{11}$.

27. $\sqrt{\frac{144}{529}} = \frac{\sqrt{144}}{\sqrt{529}} = \frac{12}{23}$.

28. $\sqrt{\frac{2}{7}} = \sqrt{.285714+} = .5345+$;

or, $\sqrt{\frac{2}{7}} = \sqrt{\frac{14}{49}} = \frac{\sqrt{14}}{7} = \frac{3.7416+}{7} = .5345+$.

The *denominator* may always be made a perfect square by introducing a factor, which factor must also be introduced into the numerator in order to preserve the value of the fraction.

29. $\sqrt{\frac{3}{4}} = \sqrt{.75} = .866+;$

or, $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} = \frac{1.732+}{2} = .866+.$

33. $\sqrt{1\frac{2}{3}} = \sqrt{\frac{5}{3}} = \sqrt{\frac{15}{9}} = \frac{\sqrt{15}}{3} = \frac{3.873-}{3} = 1.291-.$

Page 331.

14. $\dot{3}4285.\dot{7}00(32.48662+$

	27
2700	7285
180	
4	
2884	5768
307200	1517700
3840	
16	
311056	1244224
31492800	273476000
77760	
64	
31570624	252564992
31570\|6\|2\|4	20911008
	18942374
	1968634
	1894237
	74397
	63141
	11256

In extracting the cube root, when we come to annexing 0's, after having obtained a few figures of the decimal part of the root in the regular way, a few others may be obtained by mere division, dropping one order from the right of the divisor at each division.

15. 34.3472 is pointed for extracting the cube root thus: $3\dot{4}.\dot{3}47\dot{2}00$.

19. .4932 is pointed thus: $.\dot{4}93\dot{2}00$.

20. .8 is pointed thus: $.\dot{8}00$, the first figure of the root being .9.

26. $\sqrt[3]{\frac{14}{33}} = \sqrt[3]{\frac{14 \times 33 \times 33}{33 \times 33 \times 33}} = \frac{\sqrt[3]{15246}}{33} = \frac{24.79+}{33} .751+.$

Page 331, Art. 434.

1. $\sqrt{74529} = 273.$

2. $\sqrt{160} = 12.649+.$ $(12.649+) \times 16\frac{1}{2} = 208.71$ nearly.

3. $\sqrt{160 \times 40} = \sqrt{6400} = 80.$ $\sqrt{160 \times 80} = \sqrt{12800} = 113.137+.$

4. $10^a = 10$ sq. dekams. $\therefore \sqrt{10} = 3.162.$ $\sqrt{40} = 6.3245+.$ $\sqrt{60} = 7.7459+.$ $\sqrt{100} = 10.$ $\sqrt{2500} = 50.$ $\sqrt{7000} = 83.666+.$ $10^{Dm} = 1^{Hm}.$ $50^{Dm} = 5^{Hm}.$ $83^{Dm}666 = 8^{Hm}366.$

5. $\sqrt{260 \times 160} = \sqrt{41600} = 203.96+$ is the width.

6. $\sqrt{1 \times 2} = \sqrt{2} = 1.414+.$ $\sqrt{3} = 1.732+.$ $\sqrt{4} = 2.$ $\sqrt{10} = 3.162+.$ $\sqrt{9} = 3.$

Page 332, Art. 436.

1. $\sqrt{4 \times 9} = \sqrt{36} = 6$. $\sqrt{3 \times 27} = \sqrt{81} = 9$.

2. $7 : 4 :: 4 : \frac{16}{7}$ $(= 2\frac{2}{7})$.

$8 : 12 :: 12 : \frac{12 \times 12}{8}$ $(= 18)$. $12 : 8 :: 8 : \frac{8 \times 8}{12}$ $(= 5\frac{1}{3})$.

$4 : 6 :: 6 : \frac{36}{4}$ $(= 9)$. $6 : 4 :: 4 : \frac{16}{6}$ $(= 2\frac{2}{3})$.

$\frac{2}{3} : \frac{4}{5} :: \frac{4}{5} : \frac{4}{5} \times \frac{4}{5} \times \frac{3}{2}$ $(= \frac{24}{25})$. $\frac{3}{5} : \frac{2}{3} :: \frac{2}{3} : \frac{2}{3} \times \frac{2}{3} \times \frac{5}{3}$ $(= \frac{20}{27})$.

Page 332, Art. 437.

1. $\sqrt[3]{45499293} = 357$. $\sqrt[3]{487.42} = 7.87+$.

2. $\sqrt[3]{2150.4} = 12.9+$. $\sqrt[3]{215040} = 59.9+$.

3. $\sqrt{389017} = 73$.

4. $\sqrt{128} = 5.039+$.

5. There were $\frac{97.20}{.20}$, or $5 \times 97.20 = 486$ cu. yd.

Taking the depth as a measure, it would be contained in the width 2 times, and in the length 6 times. Hence the whole contents are 3×6, or 18 cubes each of whose edges are the depth. One of these cubes contains $\frac{486}{18} = 27$ cu. yd. Hence the depth is $\sqrt[3]{27} = 3$ yd. The width is, therefore, 9 yd., and the length 18 yd. In feet the dimensions are $9 \times 27 \times 54$.

6. $\sqrt[3]{2} = 1.26-$. $\sqrt[3]{3} = 1.442+$. $\sqrt[3]{4} = 1.58+$. $\sqrt[3]{8} = 2$. $\sqrt[3]{27} = 3$. $\sqrt[3]{40} = 3.42-$.

PROBLEMS IN MENSURATION.

Page 333, Art. 440.

1. $\sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} = 3.6055+$. .6055 in. $= .6055 \times 12$ ft. $= 7.266$ ft.

2. $\sqrt{(10)^2 + 4\frac{1}{2})^2} = \sqrt{100 + 20.25} = \sqrt{120.25} = 10.965+$. .965 ft. $= 11.59$ in. nearly.

3. $\sqrt{(10)^2+(15)^2}=\sqrt{325}=18.027+$. .027 ft. = .324 in.

4. $\sqrt{(18)^2+(20)^2+(12)^2}=\sqrt{324+400+144}=\sqrt{868}=29.461+$. .461 ft. = .461 × 12 = 5.532 in.

5. $\sqrt{(160)^2-(100)^2}=\sqrt{25600-10000}=\sqrt{15600}=$ 124.9 very nearly.

6. $\sqrt{(30)^2-(12)^2}=\sqrt{900-144}=\sqrt{756}=27.495+$. .495 ft. = .495 × 12 = 5.94 in.

7. $\sqrt{(16)^2\times(8)^2}=\sqrt{320}=17.888+$. .888 ft. = .888 × 12 = 10.65 in.+.

$\sqrt{(16)^2+(10\frac{2}{3})^2}=\sqrt{369.7777+}=19.229+$. .229 ft. = .229 × 12 = 2.748 in.

$\sqrt{(16)^2+(16)^2}=\sqrt{512}=22.627+$. .627 ft. = .627 × 12 = 7.524 in.

$\sqrt{(32)^2+(16)^2}=\sqrt{1280}=35.77+$. .77 ft. = .77 × 12 = 9.24 in.

Page 335, Art. 442.

1. 3.1416 × 50 = 157.08. 157.08 in. = 13 ft. 1.091 in.

2. $\frac{33\frac{1}{3}}{3.1416}=\frac{100}{9.4248}=10.61+$. .61 ft. = 7.32 in.

3. $\frac{2\times 3962\times 3.1416}{24}=1037.2516$.

Page 336, Art. 444.

1. The same as if it were cut "straight across;" i.e., $\frac{1}{2}\times 8=4$.

2. $\frac{20\times 18}{9}=40$.

3. $\frac{320\times 160}{160}=320$.

4. $\frac{15\times18\times12\times12}{4\times8}=1215.$ $\frac{15\times18\times12\times12}{2\times8}=2430.$

5. $\frac{20\times36\times12\times12}{6\times6}=2880.$

6. $\frac{1}{4}$ of $\frac{20\times24+(40+48)\times10\frac{1}{2}-[(3\times3\frac{3}{4}\times7\frac{1}{2})+(4\times6\times3\frac{1}{2})]}{9}$

$=\frac{480+88\times10\frac{1}{2}-(84\frac{3}{8}+84)}{4\times9}=\frac{480+924-168\frac{3}{8}}{36}=$

$\frac{1235.625}{36}=34.32+.$

7. 27 in. $=\frac{27}{36}=\frac{3}{4}$ yd. $\frac{12\times1\frac{1}{4}}{\frac{3}{4}}=12\times\frac{5}{4}\times\frac{4}{3}=20.$

Page 337, Art. 446.

1. $\frac{20\times24}{160}=3.$

2. $\frac{85\times75}{2\times160}=19\frac{52}{64}.$

3. $\frac{1}{2}$ the sum of the sides $=\frac{1}{2}(12+18+24)=27.$

$\sqrt{27\times15\times9\times3}=\sqrt{10935}=104.57+.$

4. $\frac{\sqrt{45\times25\times15\times5}}{160}=1.8154+.$

5. $\frac{7.52\times5.32}{2\times10}=2.00032.$

Page 338, Arts. 447, 448.

1. $\left(\frac{45+60}{2}\times50\right)\div160=16\frac{13}{32}.$

2. $\frac{12\times14\times21}{12\times12}=24\frac{1}{2}.$

3. $\frac{10 \times 12 \times 10}{12} = 100$ sq. ft.

$\frac{8 \times 16 \times 8}{12} = 85\frac{1}{3}$ sq. ft.

$\frac{20 \times 14 \times 10}{12} = 233\frac{1}{3}$ sq. ft.

$418\frac{2}{3}$ sq. ft.

$418\frac{2}{3}$
14
$1681\frac{1}{3}$
418
$5.861\frac{1}{3}$

4. $\frac{30 \times 12 \times 6 \times 5}{12 \times 4} = 225$

$\frac{40 \times 10 \times 9 \times 3}{12 \times 2} = 450$

$\frac{20 \times 6 \times 12 \times 5}{12} = 600$

1275

EXPLANATION. — The width of one of the first lot of boards is $\frac{6}{12}$ ft. Now, multiplying this by the length, we have the area in square feet; viz., $\frac{12 \times 6}{12}$. This, multiplied by the number of this kind of boards, 30, gives the square feet in this lot, $\frac{30 \times 12 \times 6}{12}$. But, as the boards are $1\frac{1}{4}$ in. thick, $\frac{1}{4}$ is added, or, what is the same thing, we multiply by $\frac{5}{4}$, obtaining $\frac{30 \times 12 \times 6 \times 5}{12 \times 4}$ ft. board meas. in the lot, etc.

5.

1st.	2	rooms,	$2 \times 18 \times 15 = 540$	sq. ft.
2d.	2	"	$2 \times 15 \times 12 = 360$	"
	1	"	$12 \times 16 = 192$	"
	1	"	$12 \times 12 = 144$	"
	2	"	$2 \times 10 \times 12 = 240$	"
	2	"	$2 \times 10 \times 10 = 200$	"

4)1676 Total sq. ft.

Add $\frac{1}{4}$ since the stuff is $1\frac{1}{4}$ in., 419

2.095 Board measure.

$18 Price per M.

16760
2095

$37.710

6.

Scantling,	27
Feet in each,	12
	.324 ft.
Cost per M,	12
	$3.888

OPERATION.

11 + 6 = 17	Lengths planks.
6	Wide.
102	Planks.
24	ft. in plank.
2.448	ft. of plank.
18	Cost per M.
19584	
2448	
$44.064	Cost of plank.
3.888	" scantling.
$47.952	Total cost.

EXPLANATION. — The front of lot is $4 \times 16\frac{1}{2}$, or 66 ft. To this add the 6 ft. which the front walk must run by the end of the side walk, and we have 72 ft., which takes 6 lengths of 12 ft. plank. Again, the side is $8 \times 16\frac{1}{2}$, or 132 ft. long, and hence takes exactly 11 lengths of 12 ft. plank. Such plank are sawed 1 ft. wide: hence it takes 6 for a length, or 102 in all. Now 1 plank 12 ft. long, 12 in. wide, and 2 in. thick, measures 24 ft. board measure, and there are 2448 ft. of plank. In order that each plank may rest on 4 stringers, there will need to be but 3 stringers to each length, as the ends of the plank will lap on to the same stringer. Hence there will be required for the front, which has 6 lengths of plank, 18 stringers, with an extra for the end, or 19 in all. For the 11 lengths of plank on the side there will be required 34 stringers. Hence in all $19 + 34 = 53$ stringers, each 6 ft. long. This will require $\frac{1}{2}$ of 53, or 27 12-ft. scantling, $\frac{1}{2}$ a scantling being waste. Each scantling measures $\frac{3 \times 4 \times 12}{12}$ $= 12$ ft. board measure; and we have $12 \times 27 = 324$ ft. of scantling.

2.448 M plank, at $18 per M	$44.064
.324 M scantling, at $12 per M . . .	3.888
Total cost	$47.952

7.

1 room 18 ft. square,	324	sq. ft.
1 " 15 × 18 ft.,	270	"
1 " 12 × 15 ft.,	180	"
1 " 16 ft. square,	256	"
	4)1030	Total sq. ft.
Add ¼, as the stuff is 1¼ in. thick,	257.5	
Lumber required,	1287.5	Bd. measure.
At $20 per M,	20	
Cost of lumber,	$25.75	

As a "square" is 100 sq. ft., there are 10.3 squares in 1030 sq. ft. This, at

75¢ per square for laying, is	$7.725
20 lb. nails, at 3¢ per lb.,	.60
Add cost of lumber,	25.75
Total cost,	$34.075

8. $\frac{(60+80)\,160}{2\times 160}=70.$

Pages 339, 340, Art. 449.

1. $\frac{(14)^2\times 3.1416}{12\times 12}=4.276+.$ $\frac{28\times 3.1416}{12}=7.33+.$

2. The areas are in the ratio $(5)^2:(7)^2$, i.e., 25 : 49 ; but, in order to carry the smoke, the larger should have 2 times the area of the smaller, if nothing is said about friction.

3. $\frac{(5\frac{1}{2})^2\times 3.1416}{4}=23.758+.$

4. $\left(\frac{160}{3.1416}\right)^2\div\frac{3.1416}{160}=\frac{160}{3.1416}=50.93.$

5. $\sqrt{\frac{160}{3.1416}}=\sqrt{50.93-}=7.13+.$

Page 340, Art. 450.

1. The room is 20 quarters wide, and hence will require 7 breadths. As each is 6 yd. long, exclusive of waste, this makes 42 yd. 6 in. waste on each of 7 breadths make 42 in., or $1\frac{1}{6}$ yd. Hence the amount required is $43\frac{1}{6}$ yd.

8 breadths $5\frac{1}{18}$ yd. long make $8 \times 5\frac{1}{18} = 40\frac{4}{9}$ yd.

2. It takes 6 breadths $\frac{172}{36}$ yd. long, or $\frac{172}{36} \times 6 = 28\frac{2}{3}$ yd. This, at 89¢ per yd., is $\$25.51\frac{1}{3}$.

Running lengthwise it takes 5 breadths $\frac{17\frac{1}{2}}{3}$ yd. long, or $\frac{175}{6}$ $= 29\frac{1}{6}$ yd. This, at 89¢ pr yd., is $\$25.95\frac{5}{6}$.

3. It is 68 ft. around the room, and, as the paper is $1\frac{1}{2}$ ft. wide, it will take $68 \div 1\frac{1}{2}$ strips; that is, 45 *whole* strips and $\frac{1}{3}$ of the width of a strip. Hence we shall have to buy 46 strips of 10 feet each, or 460 feet. Now there are 24 ft. in a roll: hence we must buy $460 = 24$, or $19\frac{1}{3}$ rolls. As they do not cut rolls, we shall have to buy 20 rolls. Deducting $\frac{1}{4}$, 15 rolls would be required. This, at 40¢ per roll, is $6.00.

4.

Two sides of 1st room	32	
" ends of " "	24	
" sides of 2d "	36	
" ends of " "	30	
Total distance around the rooms .	122	ft.
13 openings, $3\frac{1}{2}$ ft. each	$45\frac{1}{2}$	ft.
	$3)76\frac{1}{2}$	
	26	

Hence there are $76\frac{1}{2}$ ft. in width around the rooms to be covered. But as each roll makes two strips, and the paper is 18 in. wide, it covers 3 ft. Hence 26 rolls are required.

Page 341, Art. 452.

1. As $(10)^2 : (15)^2$, or $\frac{100}{225} = \frac{4}{9}$.

2. As $(6)^2 : (8)^2$, or $\frac{36}{64} = \frac{9}{16}$.

Art. 453.

1. The ratio is $(3)^3 : (2)^3$, or $\frac{27}{8} = 3\frac{3}{8}$.

2. $(\frac{6}{5})^3 \times 100 = \frac{216 \times 4}{5} = \frac{864}{5} = 172\frac{4}{5}$. $(\frac{10}{5})^3 \times 100 = 800$. $(\frac{4}{5})^3 \times 100 = \frac{64 \times 4}{5} = \frac{256}{5} = 51\frac{1}{5}$.

3 and **4.** If the men were equally "stout," the ratio of their weights would be $(70)^3 : (72)^3$. Hence the weight of the second would be $(70)^3 : (72)^3 :: 175 : 190.432+$, i.e., 190.432+ lb.

Pages 341-343, Art. 454.

1. $32 \times 5 \times 4 = 640$. $\frac{32 \times 5 \times 4}{8 \times 4 \times 4} = 5$.

3. $\frac{\overset{3}{\cancel{6}} \times \overset{37}{\cancel{148}} \times \cancel{8}}{\underset{2}{\cancel{4}} \times \cancel{4} \times \cancel{8}} = \frac{111}{2} = 55\frac{1}{2}$.

EXPLANATION. — As the wood is 6 ft. long, and the pile is 148 ft. long, the area of the base of the pile is 6×148 sq. ft. On this base, 1 ft. in height makes 6×148 cu. ft.: hence 8 ft. in height makes $6 \times 148 \times 8$ cu. ft. Now, as $4 \times 4 \times 8$ cu. ft. make a cord, $6 \times 148 \times 8$ cu. ft. make $\frac{6 \times 148 \times 8}{4 \times 4 \times 8}$ = cords, or $55\frac{1}{2}$ cords.

4. $\frac{\overset{9}{\cancel{36}} \times \cancel{4} \times \cancel{4}}{\cancel{4} \times \cancel{4} \times \underset{2}{\cancel{8}}} = 4\frac{1}{2}$.

5. Because each 8 ft. in length of the pile makes $4 \times 4 \times 8$

$= 128$ cu. ft., or 1 cord, hence 250 in length makes $\frac{250}{8}$ cords.

6. $\frac{36}{8} + \frac{\overset{21}{\cancel{42}} \times 5 \times \cancel{4}}{\cancel{4} \times \underset{2}{\cancel{4}} \times 8} + \frac{\overset{10}{\cancel{20}} \times 6 \times \cancel{4}}{\cancel{4} \times \underset{2}{\cancel{4}} \times 8} = \frac{72 + 105 + 60}{16}$

$= \frac{237}{16} = 14\frac{13}{16}$.

10. $\frac{2640 \times 3 \times 4}{27} = 1173\frac{1}{3}$.

11. $\frac{20 \times 30 \times 6\frac{1}{2}}{27 \times 5} = 28.89-$.

12. $\frac{15\frac{1}{2} \times 15\frac{1}{2} \times 20 \times .3}{144} = 10.01+$.

13. $\frac{\cancel{2} \times 3\frac{1}{4} \times \overset{325}{\cancel{650}}}{27 \times \underset{2}{\cancel{4}}} = \frac{13 \times 325}{4 \times 27} = \frac{4225}{108} = 39.12+$.

EXPLANATION. — As the ditch is $1\frac{1}{2}$ ft. wide at bottom and $2\frac{1}{2}$ ft. at top, the average width is $\frac{1}{2}$ of the sum of these extremes, or $\frac{1}{2}$ of $(2\frac{1}{2} + 1\frac{1}{2}) = \frac{1}{2}$ of $4 = 2$ ft. Then the cu. ft. of earth to be removed is the product of the length and average breadth and depth in feet; i.e., $2 \times 3\frac{1}{4} \times 650$ cu. ft. Now, as 27 cu. ft. make a cu. yd., in $2 \times 3\frac{1}{4} \times 650$ cu. ft. there are $\frac{2 \times 3\frac{1}{4} \times 650}{27}$ cu. yd.; and as the digging costs 25¢, or $\frac{1}{4}$ of a dollar, it will cost $\frac{1}{4}$ as many dollars as there are cu. yd., or $\frac{2 \times 3\frac{1}{4} \times 650}{27 \times 4}$, or \$39.12+.

14. $\frac{\overset{220}{\cancel{\$}} \times \overset{\cancel{550}}{\cancel{5250}} \times \overset{175}{\cancel{525}}}{\cancel{\$} \times \underset{3}{\cancel{27}}} = 3850$.

15. $\frac{125 \times 90 \times 30}{2500 \times 10} = 13\frac{1}{2}$.

16. $\frac{40000 \times 8 \times 4 \times 2}{8\frac{1}{2} \times 4\frac{1}{2} \times 2\frac{3}{8}} = 28180+$, or 28181 bricks.

17. $\frac{40000 \times 8 \times 4 \times 2}{8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}} = 31673+$, or 31674 bricks.

Pages 343, 344, Art. 455.

1. $(18-4)^2 = 196$. $(20-4)^2 = 256$. $(30-4)^2 = 676$.

2. $\frac{12}{16}(24-4)^2 = \frac{3}{4}$ of $400 = 300$. $\frac{3}{4}(31-4)^2 = 546\frac{3}{4}$, called 547, etc.

3. $\frac{18}{16}(36-4)^2 = \frac{9}{8}(32)^2 = 1152$.

Art. 456.

2. $\left(\frac{24+29}{2}\right)\frac{2}{3} = 5\frac{3}{3}$. $[(5\frac{3}{3})^2 \times 35] \div 144 = 75+$, called 76, etc.

Page 345.

1. $\frac{(4)^2 \times 3.1416 \times 8 \times 1728}{231 \times 31\frac{1}{2}} = 95.495+$.

$\frac{(3)^2 \times 3.1416 \times 7 \times 1728}{231 \times 31\frac{1}{2}} = \frac{(3)^2 \times 3.1416 \times 7 \times 1728 \times 2}{231 \times 63}$

$= 47.0016$.

2. $\frac{(1\frac{1}{2})^2 \times 3.1416 \times 200 \times 12 \times 60}{231} = 9 \times .0136 \times 100 \times 12 \times 30 = 4406.4$.

3. $\frac{(1)^2 \times 3.1416 \times 12 \times 1728 \times .85}{7000} = 1997.368+$.

4. $(10)^2 \times 3.1416 \times 15 = 4712.4$.

Page 346, Art. 464.

1. $\frac{50 \times 14}{2} = 350.$

2. $(200)^2 \times \frac{150}{3} = 2000000.$

3. $\frac{[(4)^2 + 4 \times 5 + (5)^2] \times \overset{.0136}{\cancel{3.141\$}} \times 7 \times 1728}{\cancel{22}1 \times 31\frac{1}{3} \times 3} =$

$\frac{(16+20+25) \times .0136 \times 7 \times \overset{864}{\cancel{1728}}}{\underset{47}{\cancel{94}}} = \frac{61 \times .0136 \times 7 \times 864}{47}$

$= 106.7536+.$

Page 347, Art. 465.

1. $(3958)^2 \times 3.1416 \times 4 = 196,862,256.7296.$
$196,862,256.7296 \times 1319\frac{1}{3} = 259,726,937,378.5856.$

2. $(1)^2 \times 3.1416 \times 4 = 12.5664.$ $\frac{1}{3}$ of $12.5664 = 4.1888.$

Art. 466.

1. $(3 \times 4)\ 4 : (3 \times 4)\ 3$, or $\frac{4}{3}$. $8 : 2$, or 4. $10 : 2\frac{1}{2}$, or 4.

2. $(6)^2 \times 6 : (2\frac{1}{2} \times 10)\ 10$, or $216 : 250$, or $\frac{108}{125}$.

Art. 467.

1. $1°$ F. $= \frac{100}{180} = \frac{5}{9}°$ C.

2. $68°$ F. is $68° - 32° = 36°$ above freezing. $\frac{5}{9}$ of $36° = 20°$. Hence the temperature is $20°$ above freezing on C., which is $20°$ above 0.

$85° - 32° = 53°$. $\frac{5}{9}$ of $53° = 29\frac{4}{9}°$.

$20°$ below 0 F. is $20° + 32° = 52°$ below freezing. $\frac{5}{9}$ of $52° = 28\frac{8}{9}°$.

3. $29° \times \frac{9}{5} + 32° = 84\frac{1}{5}°$. $30° \times \frac{9}{5} + 32° = 86°$. $-18° \times \frac{9}{5} + 32° = -\frac{2}{5}°$. $-15° \times \frac{9}{5} + 32° = 32° - 27° = 5°$.

Pages 348-351.

1. $\frac{\frac{2}{3}}{4} = \frac{1}{6}$. $3\frac{1}{2} - \frac{1}{6} = 3\frac{1}{3}$. $\frac{4}{\frac{1}{5}} = 20$. $3\frac{1}{3} + 20 = 23\frac{1}{3}$.

2. $4\frac{2}{5} = \frac{22}{5}$, and $.25 = \frac{1}{4}$.

$$\therefore \tfrac{3}{5} \text{ of } 4\tfrac{2}{5} \text{ of } .25 = \tfrac{3}{5} \text{ of } \frac{\overset{11}{\cancel{22}}}{5} \text{ of } \frac{1}{\underset{2}{\cancel{4}}} = \tfrac{33}{50}.$$

3. $1.33\frac{1}{3} = 1\frac{1}{3} = \frac{4}{3}$, $4\frac{1}{5} = \frac{21}{5}$, and $\frac{2}{\frac{3}{5}} = \frac{10}{3}$.

$$\therefore 1.33\tfrac{1}{3} \times 4\tfrac{1}{5} \div \frac{2}{\frac{3}{5}} = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{3}}} \times \tfrac{21}{5} \times \frac{\overset{3}{\cancel{10}}}{\underset{5}{\cancel{10}}} = \tfrac{42}{25} = 1\tfrac{17}{25}.$$

4. $\frac{11\frac{2}{3}}{12\frac{3}{5}} = \frac{11\frac{2}{3} \times 3 \times 5}{12\frac{3}{5} \times 3 \times 5} = \frac{175}{189}$.

$\frac{5\frac{7}{8}}{\frac{3}{16}} = \frac{5\frac{7}{8} \times 16}{\frac{3}{16} \times 16} = \frac{94}{3} = 31\frac{1}{3}$.

$\frac{\frac{2}{7}}{4} = \frac{\overset{1}{\cancel{2}}}{7 \times \underset{2}{\cancel{4}}} = \frac{1}{14}$. $\frac{1}{3} - \frac{1}{14} = \frac{11}{42}$.

$\frac{175}{189} + \frac{11}{42} = \frac{950}{1134} = 1\frac{71}{378}$.

$31 + 1\frac{71}{378} = 32\frac{71}{378}$.

REMARK. — The above is given as suggesting practical expedients in handling such expressions. The spirit of the process is to use no more figures than are absolutely necessary to effect the combinations. A careful study of such a solution, and a comparison of it with one in which the complex fractions are all reduced to simple ones, and then these fractions added, and the improper fraction resulting then reduced to a mixed number, will show the advantage of the method given. Great pains should be taken in studying these exercises to obtain economical forms of solution.

5. $5\frac{1}{2} + 3 = 8\frac{1}{2} = \frac{17}{2}$. $2 - \frac{3}{7} = 1\frac{4}{7} = \frac{11}{7}$.

$\frac{17}{2} \times \frac{11}{7} = \frac{187}{14} = 13\frac{5}{14}$.

7. $\frac{5\frac{1}{3} + 2\frac{3}{4}}{4.5 - 1\frac{1}{3}} = \frac{8\frac{1}{12}}{3\frac{1}{6}} = \frac{97}{38}$. $\frac{97}{\underset{19}{\cancel{38}}} \times \frac{\overset{2}{\cancel{4}}}{25} = \frac{194}{475}$.

REMARK, — That $4.5 = 4\frac{1}{2}$, and $6.25 = 6\frac{1}{4} = \frac{25}{4}$, and that to divide by $\frac{25}{4}$ is the same as to multiply $\frac{4}{25}$, should be seen at a glance. So, again, the addition of $5\frac{1}{3}$ and $2\frac{3}{4}$, and the subtraction of $1\frac{1}{3}$ from $4\frac{1}{2}$, should be done mentally, and without reducing the fractions to improper forms.

8. $\frac{1\frac{1}{4} \times \frac{2}{3}}{5\frac{1}{2}} \div \frac{3 - \frac{2}{3}}{\frac{1}{2} \text{ of } 7} = \frac{\frac{5}{6}}{\frac{11}{2}} \times \frac{\frac{7}{2}}{\frac{7}{3}} = \frac{5}{\cancel{3}\cancel{3}_{11}} \times \frac{\cancel{3}}{2} = \frac{5}{22}$.

9. $\frac{4\frac{1}{3} + 2\frac{1}{5} - 3}{\frac{2}{5} \text{ of } 1.25} \times (\frac{2}{3} + \frac{1}{2}) = \frac{3\frac{8}{15}}{\frac{2}{4}} \times \frac{7}{6}$

$= \frac{\overset{106}{\cancel{212}}}{45} \times \frac{7}{\cancel{6}_{3}} = \frac{742}{135} = 5\frac{67}{135}$.

10. $\frac{5.8 \div .002}{1.6} + \frac{.45}{5} = \frac{29000}{16} + .09$

$= 1812.5 + .09 = 1812.59$.

11. $\frac{1 \div .0001}{.5 \div 50} \times 400 = \frac{10000}{.01} \times 400$

$= 1000000 \times 400 = 400{,}000{,}000$.

12. $\frac{42.68 \div .002}{\frac{1}{6} \text{ of } 13} \div .8 = \frac{21340}{2.6} \div .8$

$= \frac{26675}{2.6} = 10259\frac{8}{13}$.

13. $\frac{500 \div (\frac{2}{3} \text{ of } .066)}{\frac{1}{2} \text{ of } .7 \div (4 - \frac{2}{20})} = \frac{500 \div .044}{\frac{7}{20} \div \frac{399}{100}} = \frac{\frac{500000}{44}}{\frac{5}{57}}$

$= \frac{\overset{25000}{\cancel{1425000}}}{11} \times \frac{57}{\cancel{5}} = \frac{1425000}{11} = 129545\frac{5}{11}$.

14. $\frac{.3 + .03 + .003}{3.5 \div .07} = \frac{.333}{50} = .00666$.

15. $\frac{56 \div .007}{\frac{1}{2} \text{ of } .04} \div \frac{.02}{20} = \frac{8000}{.02} \times 1000 = 400{,}000{,}000$.

16. $\frac{(\frac{2}{3} + 4.2) \div (.125 \times \frac{5}{2})}{.375 \times (\frac{3}{5} - .16\frac{2}{3})} = \frac{\frac{146}{30} \times \frac{16}{5}}{\frac{3}{8} \times \frac{13}{30}}$

$= \frac{146}{\cancel{30}} \times \frac{16}{5} \times \frac{8}{3} \times \frac{\cancel{30}}{13} = 95\frac{163}{195}$.

17. $\frac{1-\frac{1}{2}}{1+\frac{1}{2}} \times \frac{1+\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{1}{2}}{\frac{3}{2}} \times 1 = \frac{1}{3}$.

18. $\frac{.05 \times 1.02}{\frac{3}{4}} \div \left(\frac{1}{6\frac{1}{2}} - \frac{.005}{50}\right)$

$= \frac{.204}{3} \div \left(\frac{2}{13} - \frac{1}{10000}\right) = .068 \div \frac{19987}{130000}$

$= \frac{68}{1000} \times \frac{130000}{19987} = \frac{8840}{19987}$.

19. $\frac{3.7 + 1.05 + .508}{.43 - .005} \times (\frac{2}{3} - .5)$

$= \frac{5.258}{.425} \times \frac{1}{6} = \frac{2629}{1275} = 2\frac{79}{1275}$.

20. $(.0008 \div .008) \times 10000 = .1 \times 10000 = 1000$.

21. $\frac{.468}{200} \times \frac{5}{2.12\frac{1}{2}} \div 5\frac{3}{8} = \frac{\overset{39}{\cancel{234}}}{\underset{\substack{125\\625}}{\cancel{1000000}}} \times \frac{5}{6} \times \frac{\overset{2}{\cancel{4}}}{\cancel{8}} \times \frac{\cancel{8}}{43}$

$= \frac{78}{43 \times 625 \times 17} = \frac{78}{456875}$.

22. $\frac{1-.0001}{.5 \div 5} + 3\frac{1}{2} = \frac{.9999}{.1} + 3.5$

$= 9.999 + 3.5 = 13.499$.

23. $\frac{4-.002}{3 \div .03} + \frac{.04 \div .0002}{.01\frac{1}{2}} = \frac{3.998}{100} + \frac{200}{.015}$

$= .03998 + 13333.33333\frac{1}{3} = 13333.37331\frac{1}{3}$.

24. $\frac{\sqrt{.4} + \sqrt{.9}}{\sqrt{.36} + \sqrt{.16}} + \frac{\sqrt{.4 + .9}}{\sqrt{.36 - .16}}$

$= \frac{.63245553 + .9486833}{.6 + .4} + \frac{\sqrt{1.3}}{\sqrt{.2}}$

$= 1.58113883 + \sqrt{6.5}$

$= 1.58113883 + 2.54950976 = 4.13064859$.

25. $\frac{\sqrt{\frac{4}{9}} \div \sqrt{\frac{16}{25}}}{\sqrt{\frac{1}{5}} \times \sqrt{5}} = \frac{\frac{2}{3} \div \frac{4}{5}}{\sqrt{1}} = \frac{\frac{2}{3}}{1} \times \frac{5}{\cancel{4}_{2}} = \frac{5}{6}$.

26. $\sqrt{\frac{2}{3}} : \sqrt{\frac{3}{2}} :: \sqrt{7} : ?$

$\sqrt{\frac{3}{2} \times 7 \div \frac{2}{3}} = \sqrt{\frac{3}{2} \times 7 \times \frac{3}{2}} = \sqrt{\frac{9}{4} \times 7}$

$= \frac{3}{2} \times \sqrt{7} = 3.9686+.$

27. $\frac{\sqrt{4.9} + \sqrt{3.6}}{\sqrt{.25} - \sqrt{.01}} = \frac{2.21359 + 1.89737}{.5 - .1}$

$= \frac{4.11096}{.4} = 10.2774.$

28. $\frac{\sqrt{.0036} + \sqrt{490}}{\sqrt{.025} - \sqrt{.0025}} = \frac{.06 + 22.1359}{.158 - .05}$

$= \frac{22.1959}{.108} = 205.5176-.$

29. $\sqrt{16} \times \sqrt{4} = 4 \times 2 = 8$; or,

$\sqrt{16} \times \sqrt{4} = \sqrt{64} = 8.$

$\sqrt{16} + \sqrt{4} = 4 + 2 = 6$; but $\sqrt{16} + \sqrt{4}$ does not equal $\sqrt{20}$; as the latter is 4.5 nearly, and the former 6.

NOTE. — A number of these examples in radicals are designed to lead the pupil to observe practically that the square root of the product = the product of the square roots, and the square root of the quotient = the quotient of the square roots, but that a similar fact does not hold with reference to sums and differences.

Examples **30** and **31** are answered above.

32. $\sqrt{\frac{3}{7}} \times \sqrt{\frac{7}{3}} \times \sqrt{\frac{2}{5}} \times \sqrt{\frac{5}{2}} = \sqrt{\frac{3}{7} \times \frac{7}{3} \times \frac{2}{5} \times \frac{5}{2}}$

$= \sqrt{1} = 1.$

33. $\sqrt{\frac{3}{7}} + \sqrt{\frac{7}{3}} + \sqrt{\frac{2}{5}} + \sqrt{\frac{5}{2}}$ *does not* $= \sqrt{\frac{3}{7} + \frac{7}{3} + \frac{2}{5} + \frac{5}{2}}.$

The best way to solve this is to reduce the several fractions to decimals, extract the square root of each, and add the results. Thus:

$\sqrt{\frac{3}{7}} = \sqrt{.4285714285} = .65465+$

$\sqrt{\frac{7}{3}} = \sqrt{2.3333333333} = 1.52752+$

$\sqrt{\frac{2}{5}} = \sqrt{.4} = .63245+$

$\sqrt{\frac{5}{2}} = \sqrt{2.5} = 1.58113+$

$\therefore \sqrt{\frac{3}{7}} + \sqrt{\frac{7}{3}} + \sqrt{\frac{2}{5}} + \sqrt{\frac{5}{2}} = 4.39575+$

34. $5\frac{1}{2} : 3\frac{1}{4} :: \frac{2}{3} : ?$

$$(3\tfrac{1}{4} \times \tfrac{2}{3}) \div 5\tfrac{1}{2} = \frac{13 \times \cancel{2} \times \cancel{2}}{\cancel{4} \times 3 \times 11} = \tfrac{13}{33}.$$

35. $\frac{2}{5} : 1\frac{1}{5} :: 8 : ?$

$$(1\tfrac{1}{5} \times 8) \div \tfrac{2}{5} = \frac{6 \times \overset{4}{\cancel{8}} \times \cancel{5}}{\cancel{5} \times \cancel{2}} = 24.$$

36. Because, to find the 4th term of a proportion, we multiply together the 2d and 3d, and divide the product by the 1st; but to divide by the 1st is the same as to multiply by its reciprocal. Hence the 4th term is the continued product of the 2d, 3d, and reciprocal of the 1st.

37. $\sqrt{\frac{3}{4}} : \sqrt{\frac{3}{9}} :: 10 : ?$

$$10 \times \sqrt{\tfrac{3}{9} \times \tfrac{4}{3}} = 10 \times \sqrt{\tfrac{4}{9}} = 10 \times \tfrac{2}{3} = 6\tfrac{2}{3}.$$

38. $\dfrac{3 \times 1.02}{.0\ \ 5} = \dfrac{3 \times 102}{5} = \tfrac{306}{5} = 61\tfrac{1}{5}.$

39. $\dfrac{.4 \times .03}{6\frac{1}{2}} = \dfrac{4 \times .03}{65} = \tfrac{12}{6500} = \tfrac{3}{1625}.$

40. $\dfrac{.001 \times .02}{.3} = \dfrac{.00002}{.3} = .000066\tfrac{2}{3}.$

41. $\dfrac{1.00\frac{1}{2} \times .0\frac{2}{5}}{4.0\frac{3}{4}} = \dfrac{1.005 \times .04}{4.075} = \dfrac{40.2}{4075}$

$$= \tfrac{402}{40750} = \tfrac{201}{20375} = .0098\tfrac{106}{163}.$$

42. $\sqrt{2\frac{1}{2}} + \sqrt{4\frac{1}{5}} = \sqrt{2.5} + \sqrt{4.2}$

$$= 1.58114 + 2.04939 = 3.63053.$$

43. $\sqrt{2\frac{1}{2}} \times \sqrt{4\frac{1}{5}} = \sqrt{\tfrac{5}{2} \times \tfrac{21}{5}} = \sqrt{\tfrac{21}{2}}$

$$= \sqrt{10.5} = 3.2404-.$$

44. $\sqrt{16} \div \sqrt{4} = 4 \div 2 = 2.$

$\sqrt{16} - \sqrt{4} = 4 - 2 = 2.$

45. Yes: since $\sqrt{16} \div \sqrt{4} = 4 \div 2 = 2$; and

$$\sqrt{16} \div \sqrt{4} = \sqrt{4} = 2.$$

46. No: since $\sqrt{16} - \sqrt{4} = 4 - 2 = 2$; and
$\sqrt{16 - 4} = \sqrt{12} = 3.46+$.

47. $\sqrt{\frac{2}{5}} \div \sqrt{\frac{2}{5}} = \sqrt{\frac{2}{5} \div \frac{2}{5}} = \sqrt{1} = 1$; or, a quantity divided by itself is 1.

48. $\sqrt{\frac{2}{5}} - \sqrt{\frac{2}{5}} = 0$, since it is the difference between two equal quantities.

49. $\frac{\sqrt{2\frac{2}{3} \times 6}}{\sqrt{1\frac{1}{2}}} = \sqrt{\frac{2\frac{2}{3}}{1\frac{1}{2}} \times 6} = \sqrt{\frac{16}{9}} \times 6 = \frac{4}{3} \times 6 = 8.$

50. $1\frac{1}{2} : 2\frac{2}{3} :: 36 : ?$

$$\frac{36 \times 2\frac{2}{3}}{1\frac{1}{2}} = \frac{\overset{4}{\cancel{36}} \times 16}{\cancel{9}} = 64.$$

The answer to Ex. **50** is the square of that to Ex. **49**, since the terms of **50** are those of **49** squared.

51. $\frac{\sqrt{.6 \times .15}}{5} = \frac{\sqrt{.09}}{5} = \frac{.3}{5} = \frac{3}{50}$.

52. $\frac{3 \times \sqrt{1.1}}{\sqrt{.8}} = 3 \times \sqrt{\frac{1.1}{.8}} = 3 \times \sqrt{1.375}$
$= 3 \times 1.1726+ = 3.5178+$.

53. $\frac{\sqrt{\frac{9}{16}} \times \sqrt{\frac{4}{5}}}{\sqrt{\frac{2}{3}}} = \frac{\sqrt{\frac{9}{16} \times \frac{4}{5}}}{\sqrt{\frac{2}{3}}} = \sqrt{\frac{9}{\cancel{16}_{4}} \times \frac{\cancel{4}}{5} \times \frac{3}{2}}$
$= \sqrt{\frac{27}{40}} = \sqrt{.675} = .8216-$.

54. See above.

55. $\frac{3\frac{2}{5} + 5\frac{3}{4}}{5\frac{1}{2} - .025} = \frac{3.4 + 5.75}{5.5 - .025} = \frac{9.15}{5.475} = 1\frac{48}{73}$.

Or, $\frac{3\frac{2}{5} + 5\frac{3}{4}}{5\frac{1}{2} - .025} = \frac{3\frac{2}{5} + 5\frac{3}{4}}{5\frac{1}{2} - \frac{1}{40}}$ (multiplying numerator and denominator by 40) $\frac{136 + 230}{220 - 1} = \frac{366}{219} = \frac{122}{73} = 1\frac{49}{73}$.

56. $\sqrt{\frac{.004 \times 7}{.002}} = \sqrt{\frac{28}{2}} = \sqrt{14} = 3.7417-$.

57. $\sqrt{\frac{5 \times \frac{3}{5}}{10}} = \sqrt{\frac{3}{10}} = \sqrt{.3} = .5477+.$

58. $\sqrt{\frac{3}{4}} \times \sqrt{\frac{4}{3}} = \sqrt{\frac{3}{4} \times \frac{4}{3}} = \sqrt{1} = 1.$

59. $\sqrt{\frac{3}{4}} + \sqrt{\frac{4}{3}} = \sqrt{.75} + \sqrt{1.3333333\frac{1}{3}}$
$= .866 + 1.1547 = 2.0207+.$

60. $\sqrt{\frac{.005 \div .01}{3 - 2.01}} = \sqrt{\frac{.5}{.99}} = \sqrt{\frac{5}{9.9}}$
$= \sqrt{.505050+} = .7107-.$

61. $\frac{(4\frac{1}{2} - 3\frac{1}{5})\ (.2 + 3)}{.3\ (5.6 \div .07)} = \frac{1.3 \times 3.2}{.3 \times 80} = \frac{.052}{.3}$
$= \frac{52}{300} = \frac{13}{75}.$

62. $\frac{(3 - \frac{1}{3}) \div (1 - \frac{2}{5})}{.04\ (1 - .05)} = \frac{\frac{8}{3} \div \frac{3}{5}}{\frac{1}{25} \times \frac{95}{100}}$
$= \frac{8}{3} \times \frac{5}{3} \times \frac{25}{1} \times \frac{100}{95} = \frac{20000}{171} = 116\frac{124}{171}.$

63. $\frac{(4 \div .04)\ (4 - .04)}{(3 \times .03)\ (3 + .03)} = \frac{100 \times 3.96}{.09 \times 3.03} = \frac{396}{.2727}$
$= \frac{1320000}{909} = \frac{440000}{303} = 1452\frac{44}{303}.$

64. $\frac{2.08 \div (1 - \frac{1}{5})}{(2 + \frac{1}{5}) - (\frac{1}{3} \text{ of } \frac{3}{5})} = \frac{2.08 \div .8}{2.2 - .2} = \frac{2.6}{2} = 1.3.$

65. $\frac{1}{\frac{2}{3}} : ? :: \frac{\frac{3}{5}}{2\frac{1}{2}} : 4.$ $4 \times \frac{3}{2} \div \frac{\frac{3}{5}}{\frac{5}{2}} = 4 \times \frac{3}{2} \div (\frac{3}{5} \times \frac{2}{5})$
$= \frac{4 \times 3 \times 5 \times 5}{2 \times 3 \times 2} = 25.$

66. $2 \times 3\frac{1}{5} : (5 - 1\frac{1}{3}) :: ? : \frac{1}{2} \text{ of } \frac{3}{7}.$ $6\frac{2}{5} : 3\frac{5}{6} :: ? : \frac{3}{14}.$

$\frac{3}{14} \times \frac{32}{5} \div \frac{23}{6} = \frac{3 \times 32 \times 6}{14 \times 5 \times 23} = \frac{288}{805}.$

67. $(2 - \frac{3}{5}) \div \frac{4\frac{1}{2}}{\frac{3}{7} \text{ of } \frac{5}{3}} = 1\frac{2}{5} \div (\frac{3}{2} \times \frac{7}{3} \times \frac{3}{5})$
$= \frac{7}{5} \times \frac{2}{3} \times \frac{3}{7} \times \frac{5}{9} = \frac{2}{27}.$

68. $(1\frac{3}{8} - \frac{5}{8}) \div (.02 - .002) = \frac{3}{4} \div .018$

$$= \frac{\cancel{2}}{3} \times \frac{1000}{\cancel{18}_{9}} = \frac{1000}{27} = 37\tfrac{1}{27}.$$

69. $\dfrac{\frac{11}{2} \times \frac{8}{3}}{2 \div .02} \div \frac{1}{8} = \dfrac{11}{\cancel{2}} \times \dfrac{\cancel{8}}{3} \times .01 \times \dfrac{\cancel{4}}{\cancel{8}} \times \dfrac{3.52}{3}$

$$= 1\tfrac{52}{300} = 1\tfrac{13}{75}.$$

70. $\frac{4}{5} : \frac{512}{125} :: 8 : ?$ $\quad \dfrac{512 \times \overset{2}{\cancel{8}} \times 9}{125 \times \cancel{4}} = \frac{9216}{125} = 73\tfrac{91}{125}.$

71. $(\frac{11}{2})^2 \div \sqrt{2\tfrac{1}{2} \times .016} = \frac{121}{4} \div \sqrt{.04}$

$$= \tfrac{121}{4} \div .2 = \tfrac{121}{4} \times 5 = \tfrac{605}{4} = 151\tfrac{1}{4}.$$

72. $(\frac{22}{5})^3 \div (.12)^2 \times \sqrt{.4} = \dfrac{10648}{125} \times \dfrac{.63245553}{.0144}$

$$= \frac{6734.38638}{1.8} = 3741.3258+.$$

73. $(2.01)^2 + (5.3)^3 - (2.125)^2$

$$= 4.0401 + 148.877 - 4.515625 = 148.401475.$$

74. $5 \times \dfrac{\frac{3}{16}}{\frac{9}{4}} \times \dfrac{\frac{167}{12}}{\frac{21}{5}} = 5 \times \dfrac{\cancel{3}}{\cancel{16}_{4}} \times \dfrac{\cancel{4}}{\cancel{9}_{3}} \times \frac{167}{12} \times \frac{5}{21}$

$$= \tfrac{4175}{3024} = 1\tfrac{1151}{3024}.$$

75. $\dfrac{1.8973 - .4}{.4444 - .3535} \div \sqrt{\frac{3}{2} \times \frac{3}{2} \times \frac{6}{5}} = \dfrac{1.4973}{.0909} \div \sqrt{2.7}$

$$= \frac{1.4973}{.0909 \times 1.643} = 10+.$$

76. $\frac{2}{3}$ of \$5 $=$ \$$\frac{10}{3}$, and £4 $=$ \$4.866$\frac{1}{2} \times 4 =$ \$19.466. Hence the question is, What part of \$19.466 is \$$\frac{10}{3}$? \$1 is $\dfrac{1}{19.466}$, and \$$\frac{10}{3}$ is $\frac{10}{3} \times \dfrac{1}{19.466} = \dfrac{10}{58.398} = \frac{10000}{58398} = \frac{5000}{29199}$.

77. $\frac{1}{6}$ of $\frac{2}{3}$ of $1\frac{1}{4}$ yd. $= \dfrac{1}{\cancel{5}}$ of $\dfrac{\cancel{2}}{3}$ of $\dfrac{\cancel{5}}{\cancel{4}_{2}}$ yd. $= \frac{1}{6}$ yd.

$2\frac{1}{2}^{\text{m}} = \dfrac{11 \times 39.37}{5 \times 36}$ yd. Hence we have

$$\frac{1}{6} \div \frac{11 \times 39.37}{5 \times 36} = \frac{5 \times \overset{6}{\cancel{36}}}{\cancel{6} \times 11 \times 39.37} = \frac{3000}{43307}.$$

78. $\frac{1}{6}$ of $7\frac{1}{2}$ ft. $= \frac{1}{6}$ of $\frac{15}{2} = \frac{5}{4}$ft. If $\frac{5}{4}$ ft. is $\frac{3}{4}$ of a certain number of feet, $\frac{1}{3}$ is $\frac{5}{4}$ and $\frac{3}{3}$ is $\frac{9}{4}$. Now $\frac{9}{4}$ ft. $= \frac{9 \times 12}{4 \times 39.37}$m $= \frac{2700}{3937}$m.

79. $5\frac{1}{2}$Kg $= 5\frac{1}{2} \times 2.2046$ lb. $= 12.1253$ lb.

$= \frac{12.1253}{10}$, or $\frac{121253}{100000}$, of 10 lb.

80. $7\frac{1}{2}$ dg $= 1.5432 \times 7\frac{1}{2}$ gr. $= 11.574$ gr.

$= \frac{11.574}{10}$, or $\frac{11574}{10000} = \frac{5787}{5000}$, of 10 gr.

81. 4*s*. 6*d*. $= £\frac{4\frac{1}{2}}{20} = £\frac{9}{40} = \frac{9}{40} \times \frac{4.8665}{3.86}$Nap,

or $\frac{9 \times 4.8665}{40 \times 3.86 \times 2} = \frac{87597}{617600}$, of 2Nap.

82. $5\frac{2}{3}$ M $= \$5\frac{2}{3} \times .238$, or $\frac{5\frac{2}{3} \times .238}{2\frac{1}{2}} = \frac{17 \times \overset{119}{238}}{\underset{3750}{7500}}$

$= \frac{2023}{3750}$, of $\$2\frac{1}{2}$, or a little more than $\frac{1}{2}$ of it.

83. $\frac{\frac{1}{2} \text{ of } 8\frac{2}{5}}{14\frac{7}{10}} = \frac{4.2}{14.7} = \frac{42}{147} = \frac{2}{7}$.

$\therefore\ 14\frac{2}{7} - \frac{\frac{1}{2} \text{ of } 8\frac{2}{5}}{14\frac{7}{10}} = 14\frac{2}{7} - \frac{2}{7} = 14$.

Again, $\frac{2}{3}$ of $\frac{7}{9} = \frac{14}{27}$. Now, if 14 is $\frac{14}{27}$, 1 is $\frac{1}{14}$ of $\frac{14}{27} = \frac{1}{27}$; and, if 1 is $\frac{1}{27}$ of the number, the number is $1 \div \frac{1}{27} = 27$.

84. $1\frac{1}{3} + 1\frac{1}{2} + 2 = 4\frac{5}{6}$. Hence we have

$4\frac{5}{6} : 1\frac{1}{2} :: 7\frac{1}{2} : ?$ $\quad\therefore\ \frac{7\frac{1}{2} \times 1\frac{1}{2}}{4\frac{5}{6}} = \frac{15 \times 4}{29} = 2\frac{2}{29}$.

$4\frac{5}{6} : 1\frac{1}{2} :: 7\frac{1}{2} : ?$ $\quad\therefore\ \frac{7\frac{1}{2} \times 1\frac{1}{2}}{4\frac{5}{6}} = \frac{15 \times 9}{58} = 2\frac{19}{58}$.

$4\frac{5}{6} : 2 :: 7\frac{1}{2} : ?$ $\quad\therefore\ \frac{7\frac{1}{2} \times 2}{4\frac{5}{6}} = \frac{15 \times 6}{29} = 3\frac{3}{29}$.

85. $\frac{2}{3} + \frac{3}{5} + 2.1 = \frac{20 + 18 + 63}{30} = \frac{101}{30}$.

$\frac{101}{30} : \frac{20}{30} :: 12 : ?$ $\quad\therefore\quad \frac{240}{101} = 2\frac{38}{101}$.

$\frac{101}{30} : \frac{18}{30} :: 12 : ?$ $\quad\therefore\quad \frac{216}{101} = 2\frac{14}{101}$.

$\frac{101}{30} : \frac{63}{30} :: 12 : ?$ $\quad\therefore\quad \frac{756}{101} = 7\frac{49}{101}$.

86. $2.5 + 1.1 + .5 + 3.2 = 7.3$.

$7.3 : 2.5 :: 9 : ?$ $\quad\therefore\quad \frac{2.5 \times 9}{7.3} = 3\frac{8}{73}$.

$7.3 : 1.1 :: 9 : ?$ $\quad\therefore\quad \frac{1.1 \times 9}{7.3} = 1\frac{26}{73}$.

$7.3 : .5 :: 9 : ?$ $\quad\therefore\quad \frac{.5 \times 9}{7.3} = \frac{45}{73}$.

$7.3 : 3.2 :: 9 : ?$ $\quad\therefore\quad \frac{3.2 \times 9}{7.3} = 3\frac{62}{73}$.

87. $\frac{2}{3}$ of $6 = 4$; $\frac{1}{5}$ of $20 = 4$; $\frac{1}{7}$ of $14 = 2$; and $4 + 4 + 2 = 10$. Hence we have

$10 : 4 :: \$1250 : \$500.$
$10 : 4 :: \$1250 : \$500.$
$10 : 2 :: \$1250 : \$250.$

88. $2 \times 60 = 120$; $3 \times 50 = 150$; $4 \times 35 = 140$; $7 \times 16 = 112$; and $120 + 150 + 140 + 112 = 522$. Hence we have

$522 : 120 :: \$3800 : ?$
$522 : 150 :: \$3800 : ?$
$522 : 140 :: \$3800 : ?$
$522 : 112 :: \$3800 : ?$

89. 1st. The root of a proper fraction is a proper fraction, since the numerator, being smaller than the denominator, if resolved into the same number of factors as the denominator, one of the factors of the numerator will be less than one of the factors of the denominator. But one of the factors of the numerator divided by one of the denominator is the root. Again, to extract the root of a number is to reject, or

divide, by all the equal factors except one. This divisor, in the case of a proper fraction, is less than 1. But to divide any number by a number less than 1 increases it.

90. To involve a number is to multiply it by itself. But to multiply any number by a proper fraction diminishes it.

91. The reasons are analogous to those given above for the case of a proper fraction.

92. $\frac{2}{3}$ of $3\frac{3}{4}$ gal. $=$ $\frac{2}{3}$ of $\frac{15}{4}$ of 231 cu. in.
.5 bu. $=$ $\frac{1}{2}$ of 2150.4 cu. in.

To find what part one number is of another, we divide the former by the latter.[1] Hence we have

$$\frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{4}}} \text{ of } 231 \div \frac{\overset{1075\cdot 2}{\cancel{2150.4}}}{\cancel{2}} = \frac{5 \times 231}{2 \times 1075.2} = \frac{1155}{2150.4} = \frac{5775}{10752}.$$

93. $2.6^{l} = \frac{2.6 \times 1.0567}{4}$ gal.; and
$\frac{2}{3}$ of 5 gal. $=$ $\frac{10}{3}$ gal. Hence we have
$$\frac{2.6 \times 1.0567}{4} \times \tfrac{3}{10} = \tfrac{412113}{2000000}.$$

94. .4 mm $=$.4 $\times$.03937 in.; and we have
$$\frac{.4 \times .03937}{.01} = 1.5748, \text{ or } 1\tfrac{1437}{2500}.$$

95. $25^{Dl} = \frac{9.08 \times 25}{32}$ bu. Hence we have
$$\frac{9.08 \times 25}{32 \times 10} = \frac{22.7}{32} = \tfrac{227}{320} \text{ of } 10 \text{ bu.}$$

96. $3.2^{m} = \frac{\overset{8}{\cancel{3.2}} \times 39.37}{16\frac{1}{2} \times \underset{3}{\cancel{12}}}$ rd. $= \frac{31.496}{49.5}$
$$= \tfrac{31496}{49500} = \tfrac{7874}{12375}.$$

[1] The analysis should be familiar. Thus, to find what part 5 is of 6. 1 is $\frac{1}{6}$ of 6, and 5 is 5 times as great a part as 1 is. Hence 5 is $\frac{5}{6}$ of 6. In the above example, 1 is $\frac{10}{10752}$ of $\frac{1}{2}$ of 2150.4 cu. in., etc.

97. $1.6^m = 1.6 \times 39.37$ in.; and 5 ft. $10\frac{1}{2}$ in. $= 70\frac{1}{2}$ in.
Hence 1.6^m : 5 ft. $10\frac{1}{2}$ in. :: 1.6×39.37 in. : $70\frac{1}{2}$ in.
Or the ratio is $\frac{1.6 \times 39.37}{70.5} = \frac{15748}{17625}$.

98. $2\,3 : 4^s :: 2 \times 3 \times 20$ gr. : 4×15.432 gr.; or,

$$\frac{\cancel{2} \times \cancel{3} \times \overset{5}{\cancel{20}}}{\cancel{4} \times \underset{\substack{7.716\\2.572\\.5144}}{\cancel{15.432}}} = \frac{1}{.5144}.$$

Example 99.

$\frac{3}{7}$ of $4\frac{1}{2} \times \frac{9\frac{3}{4}}{\frac{13}{20}} \times \frac{1}{15}$ of $\frac{7}{9} = \frac{\cancel{3} \times \cancel{9} \times \overset{3}{\cancel{39}} \times \overset{5}{\cancel{20}} \times \cancel{7}}{\cancel{7} \times 2 \times \cancel{4} \times \cancel{13} \times \underset{3}{\cancel{15}} \times \cancel{9}} = \frac{3}{2}$.

$\frac{3}{4}$ of £43 18*s*. $11\frac{1}{2}d. =$ £65 18*s*. $5\frac{1}{4}d. =$ \$320.81.

$3\frac{3}{9} \times \frac{1}{17\frac{1}{2}}$ of .56 of $1.75 \times 6\frac{1}{2} = \frac{\cancel{35} \times \cancel{2} \times \overset{7}{\cancel{14}} \times 7 \times 13}{9 \times \cancel{35} \times \underset{50}{\cancel{100}} \times \cancel{4} \times \cancel{2}} = \frac{237}{450}$.

$\frac{237}{450}$ of \$97.18 = \$137.56.
Hence the difference is \$320.81 — \$137.56 = \$183.25.

Example 100.

$\frac{2}{3}$ of $4\frac{1}{2} \times 7\frac{1}{5} \times \frac{9}{19\frac{1}{2}} \times \frac{5}{9} = \frac{2 \times 9 \times 39 \times 18 \times 5}{3 \times 2 \times 5 \times 39 \times 9} = 6$.

3 oz. 4 dr. 2 scr. 5 gr. $\times$ 6 = 1725 gr. $\times$ 6 = 10350 gr.
Again, $\frac{6}{11}$ of .63 of $2\frac{37}{42} \times \frac{3}{13}$ of $6\frac{1}{2}$

$$= \frac{6 \times 63 \times 121 \times 3 \times 13}{11 \times 100 \times 42 \times 13 \times 2} = \frac{297}{400}.$$

7 lb. 3 oz. Av. $= 7\frac{3}{16} \times 7000$ gr. $= 50312.5$ gr.
$\frac{297}{400}$ of 50312.5 gr. $= 74714.0625$ gr.
Finally, $\frac{10350}{74714.0625} = .1385+$.

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1. 5 is $\frac{1}{6}$ of 25. $\frac{1}{6}$ of 100 is 20. $\therefore$ 5 is 20% of 25.

2. If 11 is 6%, or $\frac{6}{100}$ of the number, $\frac{1}{100}$ is $\frac{1}{6}$ of it, or $\frac{11}{6}$, and $\frac{100}{100}$ of it is 100 times $\frac{11}{6}$, or $\frac{1100}{6} = 183\frac{2}{3}$.

3. The note had run 9 mo. 29 da. when collected, and hence amounted to \$1066.44. $\frac{3}{4}$ (75%) of this was paid. $\frac{3}{4}$ of \$1066.44 = \$799.83. The broker charged $\frac{1}{2}$%. $\frac{1}{2}$% of \$799.83 = \$4.00. Hence I received \$799.83 − \$4.00 = \$795.83.

4. $\frac{.0\$ \times .0\$}{\$\$0} = \frac{.07}{5} = .014$, the discount on \$1.

Hence the avails of \$1 would be \$1.00 − \$0.014 = \$0.986, and to obtain \$200 my note must be made for $\frac{\$200}{.986} = \$202.84$.

5. Amount of note July 1, 1876, \$534.25. Deducting payment, there remains \$534.25 − \$150 = \$384.25. Amount of this balance March 10, 1877, \$400.19. Brokerage, $\frac{3}{4}$% of \$400.19 = \$3. Amount remitted, \$400.19 − \$3 = \$397.19.

6. \$2593.75 ÷ 1.0375 = \$2500, the amount to be invested in land. Hence \$2500 ÷ 125 = 20 is the number of acres bought.

7. For \$85 I get \$3 midyear, and \$3 at the end of the year. At 10% per annum the midyear \$3 amounts to \$3.15 at the close of the year. Hence $\frac{6.15}{85} = .07\frac{4}{17}$, or $7\frac{4}{17}$%.

8. 13950 × .6 × \$0.193 = \$1615.41.

9. I sell the goods at $1\frac{1}{2}$ times \$2000 = \$3000. 3% for selling and 5% loss makes 8% out of the \$3000, or \$3000 − \$240 = \$2760. Hence I make \$2760 − \$2000 = \$760 on \$2000, or $\frac{\$760}{2000} = \$0.38$ on \$1; i.e., 38%.

10. $\frac{1.00}{.06} = 16\frac{2}{3}$. $16\frac{2}{3}$ yr. = 16 yr. 8 mo. $\frac{1.00}{.07} = 14\frac{2}{7}$. $14\frac{2}{7}$ yr. = 14 yr. 3 mo. 13 da.

11.

1.00	
.05	
1.05	Amt. 1 yr.
.0525	
1.1025	" 2 yr.
.055125	
1.157625	" 3 yr.
.05788125	
1.21550625	" 4 yr.
.0607753125	
1.2762815625	" 5 yr., etc.

Proceeding thus, or from a table, we find that in 14 yr. \$1 at 5% compound interest amounts to \$1.9799316. There is then \$2 — \$1.9799316, or \$0.0200684 to gain. In 1 yr. \$1.9799316 gains \$0.09899658. Hence to gain \$0.0200684 requires $\frac{200684}{9899658} = .2027+$ yr. = 2 mo. 13 da. Hence the time is 14 yr. 2 mo. 13 da.

12.

Prin.	Time.	Interest.	Payments
\$5600	4 mo. 18 da.	\$119.466	\$500
5219.466	3 " 16 "	92.21	200
5111.676	3 " 26 "	98.826	300
4910.504	3 " 16 "	86.75	150
4847.256	8 " 24 "	213.28	5060.54

The note ran 2 yr.

Amt. of note for 2 yr. at 7%	\$6272.00
" \$500 payt., 1 yr. 7 mo. 22 da.	\$549.33⅓
" \$200 " 1 yr. 4 mo. 6 da.	216.20
" \$300 " 1 yr. 10 da.	318.50
" \$150 " 8 mo. 24 da.	156.60
Total amt. of payments	\$1240.63
Balance due	\$5031.37

17. $\$202.50 \div .015 = \$13500$, which is $\frac{3}{4}$ the value of the warehouse. Hence $\frac{\$13500}{3} \times 4 = \$18000$.

18. Of every \$1 levied, $\$1.00 - \$0.03 = \$0.97$ is collected. But of this the collector receives 5%. 5% of \$0.97 is \$0.0485. Hence there remains to apply on the \$50000 only $\$0.97 - \$0.0485 = \$0.9215$. To raise \$50000, therefore, $\$50000 \div .9215 = \$54259.36$ must be levied. Of this a man whose property is assessed at \$3500 will be assessed $\frac{3500}{2000000} = \frac{7}{4000}$. Now, $\frac{7}{4000}$ of $\$54259.36 = \$94.95$.

19. $\frac{294000}{70000000} = .0042$. $\$70000000 \times .0002 = \$14000$.

20. $\$3 + \$3 + .03$ of $\$3 = \$6.09$. Hence \$75 yields \$6.09 per ann. $\frac{600}{75} = 8\frac{3}{25}$.

21. If I sold $\frac{1}{2}$ the stock at $\frac{2}{3}$ the cost of the whole, I should have sold the whole at $\frac{4}{3}$, or $1\frac{1}{3}$ times the cost; that is, I made $\frac{1}{3}$ the cost, or $33\frac{1}{3}\%$.

22. What he bought at 75 he sold at 120; i.e., he made 45 on 75. $\frac{45}{75} = \frac{3}{5}$. If \$1920 is $\frac{3}{5}$ of what he invested, $\frac{1}{5}$ is $\frac{1}{3}$ of \$1920, or \$640, and $\frac{5}{5}$ is \$3200.

23. I was selling what cost \$1.00 at \$1.25. Marking down 10% on this left the selling price $\$1.25 - \$0.125 = \$1.125$; i.e., after marking down, I make \$0.125 on \$1.00, or $12\frac{1}{2}\%$.

24. I was selling what cost \$1.00 at \$1.20. Marking up 5% on this made the selling price $\$1.20 + \$0.06 = \$1.26$. Hence I sold for \$1.26 what cost me \$1.00, and made 26¢ on \$1.00, or 26%.

25. $\$707 \div 1.01 = \$700$, face of draft. $\frac{700}{4} = 175$, number of shares.

26. Cost of cotton, $230 \times 450 \times .10\frac{3}{4} = \$11126.25$. The present worth of this at 8% discount is $\$11126.25 \div 1.08 = \$10302.08$. Hence the gain was $\$12000 - \$10302.08 = \$1697.92$.

27. $\$9.00 \div 1.125 = 8$. $\therefore$ \$8.

28. He must sell it at $\$150 + \$30 = \$180$. But this is only 95¢ on \$1.00 of what it was marked. Hence it was marked at $\$180 \div .95 = \$189.47+$.

29. If he makes $\frac{1}{4}$, he must sell at $\frac{5}{4}$ cost. If \$1.20 is $\frac{5}{4}$, the cost is \$0.96. Buying at \$0.96 and selling at \$1.28, one makes \$0.32, or $\frac{32}{96} = \frac{1}{3}$, or $33\frac{1}{3}\%$.

30. If $\frac{2}{3}$ the selling price = once the cost, $\frac{1}{3}$ the selling price = $\frac{1}{2}$ the cost, and $\frac{3}{3}$ the selling price = $\frac{3}{2}$, or $1\frac{1}{2}$ the cost. Hence, as the selling price is $1\frac{1}{2}$ times the cost, the gain is 50%.

31. 75% is $\frac{3}{4}$. Hence, after the addition, $\frac{3}{4}$ is alcohol and $\frac{1}{4}$ water. But there is 1 gal. of alcohol, and, as there is $\frac{1}{3}$ as much water, there is $\frac{1}{3}$ gal. of water.

32. The main question is, "What principal put at compound interest at 6% yields \$750 interest in 15 yr.?" To this must be added \$750 to build the first fence. Now, from the table (p. 271), we find that \$1 amounts to \$1.790848 in 10 yr. Carrying forward the computation, we find that in 15 yr. it amounts to \$2.396558; i.e., \$1.00 yields \$1.396558 int. in 15 yr. at 6%. Hence there will be required $\$750 \div 1.396558 = \$537.03$ to meet the expense after the first fence has been built; or, in all, $\$537.03 + \$750 = \$1287.03$.

33. $\sqrt{2916} = 54$.

34. $\sqrt{32 \times 128} = 64$.

35. $\sqrt{160 \times 160} = 160$, the number of rods the lot measures on one side.

$160 \times 4 = 640$, the number of rods around the lot.

$640 \times \$4 = \$2560$, the cost of the fence.

36. $$\frac{(4 \times 58 \times 5) + (4 \times 70 \times 5\frac{1}{2}) + (4 \times 65 \times 6)}{4 \times 4 \times 8} \times 5\frac{1}{2}$$

$$= \frac{1065 \times 11}{64} = \frac{11715}{64} = 183.046+.$$

37. By **458** the ratio of the 1st to its increased size is $(4)^3 : (5)^3$, or $64 : 125$. Hence the increase is $\frac{125 - 64}{64}$, or $\frac{61}{64}$ of itself.

2d. $(2)^3 : (3)^3$, or $8 : 27$. Increased $\frac{27 - 8}{8} = \frac{19}{8} = 2\frac{3}{8}$ of itself.

3d. $(1)^3 : (2)^3$, or $1 : 8$. Increased $\frac{8 - 1}{1} = 7$ times itself.

38. Increasing two dimensions 10% makes the ratio $(10)^2 : (11)^2$, (Art. **452,**) or $100 : 121$. The increase is therefore $\frac{21}{100}$ of itself. The principles of the other questions are involved in this and the preceding.

39. $\frac{3}{8}$ mi. $= \frac{3}{8}$ of 320 rd. $= 120$ rd. $\frac{1}{4}$ m. $= \frac{1}{4}$ of 320 rd. $= 80$ rd. $\sqrt{(120)^2 + (80)^2} = \sqrt{14400 + 6400} = \sqrt{20800} = 144.222+$. $200 - 144.222+ = 55.778-$.

40. $\sqrt{(30)^2 - (12)^2} = \sqrt{756} = 27.495+$. .495 ft. $= .495 \times 12 = 5.94$ in.

41. $\sqrt{(12)^2 + (15)^2} = \sqrt{369} = 19.21=$, length of rafter. $19.21 \times 2 \times 40 = 1536.8$ sq. ft. in surface of roof. As each shingle covers $6 \times 4 = 24$ sq. in., it takes $\frac{144}{24} = 6$ shingles to cover a foot. Hence there are required $6 \times 1536.8 = 9220.8$ shingles, or $9\frac{1}{4}$ M within 30 shingles.

42. Suppose 4 posts 8 ft. high supporting the deck and standing on joists at the level of the plates. To give $\frac{1}{3}$ pitch, each post must be $\frac{3}{2}$ of its own length distant from the plates, and hence is 12 ft. from the plates. Hence the deck will be $36 - 2 \times 12 = 12$ ft. wide, and $42 - 24 = 18$ ft. long. The side and end rafters will be $\sqrt{8^2 + (12)^2} = 14$ ft. 5 in., and the corner rafters $\sqrt{8^2 + 2\,(12)^2} = 18$ ft. 9.1 in. long.

43. 30 mi. per hour is $\frac{1}{2}$ mi., or 2640 ft., per minute. $\frac{2640}{6 \times 3.1416} = \frac{440}{3.1416} = 140\frac{20}{357}$.

44. The volumes of spheres being as the cubes of their diameters, we have $(1)^3 : (3)^3 :: 100 : 2700$.

45. Conceive the square drawn in such a circle, and two radii drawn to the extremities of one side. These radii form the sides of a right-angled triangle, of which the hypothenuse is the side of the required square. Hence we have $\sqrt{2\ (18)^2} = 18\sqrt{2} = 18 \times 1.4142 = 23.4556$. The area of the square, $(18\sqrt{2})^2 = (18)^2 \times 2 = 648$.

46. Conceive the cube inscribed in the sphere, and a diagonal drawn in one of its faces. The square of this diagonal equals the sum of the squares of two edges, or twice the square of one edge. Now this diagonal and one of the other edges of the cube make the sides of a right-angled triangle, of which the diameter of the sphere is the hypothenuse. Hence 3 times the square of one of the edges of the cube is the square of the diameter of the sphere; or $\frac{1}{3}$ the square of the diameter (i.e., $\frac{1}{3}$ of $4 = \frac{4}{3}$) is the square of one of the edges of the cube, and an edge of the cube is $\sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = 1.154+$.

47. $(4)^3 : (6)^3 :: 20 : 67\frac{1}{2}$.

48. The sheets cover $11\frac{1}{2}$ in. by $16\frac{1}{2}$ each. Hence 1 box covers $\frac{11\frac{1}{2} \times 16\frac{1}{2} \times 100}{144}$ sq. ft. According to the note preceding, the area of the dome is $2 \times 3.1416 \times (10)^2$. Hence we have $2 \times 3.1416 \times (10)^2 \div \frac{11\frac{1}{2} \times 16\frac{1}{2} \times 100}{144}$ boxes $= 2 \times 3.1416 \times (10)^2 \times \frac{144}{11\frac{1}{2} \times 16\frac{1}{2} \times 100} = 4.768+$, or a little more than $4\frac{3}{4}$ boxes.

49. $\frac{(30+50)\ 40}{2} = 1600$ sq. rd. $= 10$ A.

50. $2\sqrt{\frac{\frac{1}{4} \text{ of } 100}{3.1416}} = \frac{2 \times 5}{\sqrt{3.1416}} = \frac{10}{1.7724} = 5.642+.$

51. Since the volume is 4 times the square of the radius $\times$ 3.1416 $\times$ $\frac{1}{3}$ the radius, it is $\frac{4}{3} \times 3.1416 \times (\text{radius})^3$. Hence we have $\sqrt[3]{\frac{150}{\frac{4}{3} \times 3.1416}} = \sqrt[3]{\frac{37.5}{1.0472}} = 6.59+.$

52. $\frac{(852573)^3}{(7916)^3} = 1{,}249{,}330-.$

53. As $(3)^3 : (11)^3$, or as 27 : 1331.

54. As the areas of circles are in the ratio of the squares of their diameters, we have

$$\sqrt{\tfrac{2}{3} \text{ of } 2^2} = 1.633+.$$
$$\sqrt{\tfrac{1}{3} \text{ of } 2^2} = 1.154+, \text{ etc.}$$

55. As the volumes of spheres are to each other as the cubes of their diameters, we have $1 : \frac{3}{4} :: (4)^3 :$ cube of diameter of a ball $\frac{3}{4}$ as large as first. Hence

$$\sqrt[3]{\tfrac{3}{4} \times (4)^3} = \sqrt[3]{12} = 3.634+$$

For one $\frac{1}{2}$ as large we have $1 : \frac{1}{2} :: 4^3 :$ cube of the diameter, or $\sqrt[3]{\frac{1}{2} \times (4)^3} = \sqrt[3]{32} = 3.175-.$

For one $\frac{1}{4}$ as large, $1 : \frac{1}{4} :: (4)^3 : —$, or $\sqrt[3]{\frac{1}{4} \times (4)^3} = \sqrt[3]{16} = 2.52-.$

When the diameter is reduced to 2 in., its ratio to the original ball is $(2)^3 : (4)^3 :: 1 : 8$, or the ball is $\frac{1}{8}$ as large as at first.

56. Similar solids are to each other as the cubes of their like dimensions. Hence $(2)^3 : (6)^3 :: 16 : 432$, the weight of a similarly formed man 6 ft. high.

57. $\sqrt[3]{\frac{3000}{2 \times 3 \times 4}} = 5$. $\therefore$ The dimensions are $5 \times 2 = 10$, $5 \times 3 = 15$, and $5 \times 4 = 20$.

58. It takes $\frac{20 \times 12}{27} = \frac{80}{9}$, or 9 strips, each $25 \times 12 + 4 = 304$ in., or $\frac{304}{36} = 8\frac{4}{9}$ yd. long. $9 \times 8\frac{4}{9} = 76$ yd. in all.

59.

$$\frac{24 \times 24 \times 3}{4} = 432$$

$$\frac{26 \times 26 \times 3}{4} = 507$$

$$\frac{36 \times 36 \times 3}{4} = 972$$

$$\frac{28 \times 28 \times 3 \times 3}{4} = 1764$$

$$\frac{26 \times 26 \times 7 \times 2}{8} = 1183$$

$$\frac{31 \times 31 \times 7 \times 3}{8} = 2523$$

$$\frac{16 \times 16 \times 7}{8} = 224$$

7605 sq. ft.

10

\$76.05

60. $\frac{21 + 25}{2} = 23$ in. av. diam. $\frac{2}{3}$ of $23 = \frac{46}{3}$, side of square timber (by Scribner's method).

$$\frac{46 \times 46 \times 15}{3 \times 3 \times 144} = 24.5-\text{ cu. ft.}$$

$$\frac{56 \times 56 \times 20}{3 \times 3 \times 144} = 48.4-\text{ cu. ft.}$$

$$\frac{66 \times 66 \times 32}{144} = 968-\text{ cu. ft.}$$

1040.9− cu. ft

62. 3 oz. 3 pwt. 8 gr. $= 3\frac{1}{6}$ oz. $6 \times 3\frac{1}{6}$ oz. $= 19$ oz.

$\$2.25 \times 19 = \$42.75$ Cost of spoons.
$\$1.25 \times 14 = \underline{17.50}$ " chain.
$\$60.25$ Total cost.

63. $32^{l} = 32 \times .908 = 29.056$ dry qt.
1 bush. $= \underline{32.000}$ qt.
2.944 Diff.

Reckoning the liter as 1.06 liquid quarts (**291**), $1\frac{1}{4}$ hektoliters $= 1\frac{1}{4} \times 106$ qt. $= 132.5$ qt.

132.5 qt. — 126 qt. (1 bbl.) $= 6.5$ qt.

64. $\dfrac{[(8)^2 + (8 \times 10) + (10)^2]\ \overset{.0186}{\$.141\not{0}} \times 1\overset{6}{\not{0}}}{23\not{1} \times \$} = 19.9104.$

67. Weight of 2 common $\frac{1}{2}$ dollars $25^{g} = 15.432 \times 25 = 385.8$ gr. 420 gr. — 385.8 gr. $= 34.2$ gr. $\frac{342}{3858} = \frac{171}{1929}$, or about 9% (8.8%+).

68. $3.2 \times 1 \times 2 = 6.4$ cu. m. In 6.4 cu. m. there are

$$\frac{(39.37)^3 \times 6.4}{2150.4} = 181.62+ \text{ bu.}$$

Since a liter is a cubic decim, we have but to find how many cubic decims there are in the bin in order to know how many liters it contains. $32 \times 10 \times 20 = 6400$ cu. decims $= 64^{\text{hl}}$.

$$\frac{181.62 \times 60}{2.2} = 4953.3-^{\text{kg}}.$$

2.2 lb. is the legalized equivalent of a kilog (**291**).

69.

	81	47	
	71	4	9
	10	42	51
			4
	42	51	24
5	00	00	
4	17	8	36

70. $4)\overline{52 \quad 18}$

$13 \quad 4\frac{1}{2}$.

71. $\frac{12\frac{1}{2} \times 51}{4} = 159\frac{3}{8}$.

72. $1^{m} = 100^{l}$. Water weighs 1^{Kg} to a liter. Hence 100^{l} of water weighs 100^{Kg}. But as milk is 1.032 as heavy, 100^{l} of milk weighs $103^{Kg}2$.

73. $\frac{101.8}{100} = 1.018$.

74. 1 ml of water weighs $.001^{Kg} = 10$ dg. $\frac{18.42}{10} = 1.842$.

75. 2^{Hl} of water weighs 200^{Kg}. Hence $.94 \times 200 = 188^{Kg}$ is the weight of 2^{Hl} of linseed-oil.

76. $\frac{8}{10} = .8$, as 1^{l} of water weighs 10^{Hg}.

77. $\frac{4 \times (6)^2 \times 3.1416 \times 6}{3} = 904.7808$.

$\frac{4 \times (4\frac{1}{2})^2 \times 3.1416 \times 4\frac{1}{2}}{3} = \frac{381.7044}{523.0764}$.

$\frac{523.0764 \times 7\frac{1}{4}}{27.69} = 136.955+$. (See text-book, **287.**)

78. $\frac{4 \times (5)^2 \times 3.1416 \times 5 \times 7\frac{1}{4}}{3 \times 27.69} = 137.092+$.

80. As in Art. **465,** Ex. 1, the volume of the earth is found to be 259,726,937,378.5856 cu. mi.

Now the weight of 1 cu. mi. of earth, of specific gravity $2\frac{1}{2}$, is $\frac{(5280)^3 \times 1728 \times 2\frac{1}{2}}{27.69 \times 2000} = 15,129,923,305.163$ tons.

Hence the weight of the earth would be

$259,726,937,378.5856 \times 11,482,397,122.426 =$
$2,982,287,838,372,389,193,193.92$ tons.

81. The time past noon being $\frac{2}{3}$ of the time to midnight, the whole 12 hr. is divided into 5 parts, of which 2 are the time past noon, and 3 the time from noon to midnight. $\frac{2}{5}$ of 12 hr. $= 4$ hr. 48 min.

82. The man and wife eat $\frac{1}{8}$ of the quantity per day, and the wife eats of this $\frac{1}{27}$. Hence the man eats $\frac{1}{8} - \frac{1}{27}$, or $\frac{2}{27}$ per day. Therefore it will last the man $\frac{27}{2} = 13\frac{1}{2}$ da.

83. $9 : 4 :: 6 : \frac{4}{9}$ of 6, or $2\frac{2}{3}$.

84. Had he worked 100 da., he would have received \$250. As it was, he received \$190. Hence he lost by idleness \$60. But each day he was idle he lost his wages, \$2.50, and his forfeit, 50¢; i.e., \$3. Hence he was idle 20 da.

86. A and B do $\frac{1}{6\frac{2}{3}} = \frac{9}{60}$ in 1 da.

B and C " $\frac{1}{12} = \frac{5}{60}$ "

A and C " $\frac{1}{10} = \frac{6}{60}$ "

Hence A and B, B and C, and A and C, or the 3 men working 2 da. each, can do $\frac{9+5+6}{60} = \frac{20}{60}$ in 1 da. Therefore A, B, and C can do $\frac{10}{60}$ in 1 da.

But B and C can do $\frac{5}{60}$. Hence A can do $\frac{5}{60}$, or $\frac{1}{12}$, in 1 da., or the whole in 12 da.

B can do $\frac{10}{60}$ — what A and C can do, or $\frac{6}{60}$; i.e., $\frac{4}{60} = \frac{1}{15}$ in 1 da., and the whole in 15 da.

C can do $\frac{10}{60} - \frac{9}{60} = \frac{1}{60}$ in 1 da., or the whole in 60 da.

87. At the first there was in the mixture 94 oz. gold and 6 oz. silver. Hence to $\frac{2}{5}$ oz. of silver there is $\frac{1}{15}$ of 94 oz. $= \frac{94}{15}$ oz. gold, as $\frac{2}{5}$ is $\frac{1}{15}$ of 6. Then, to make 10 oz. of gold, to this $\frac{2}{5}$ oz. silver there must be added 10 oz. $- \frac{94}{15}$ oz. $= \frac{56}{15}$ oz. But this is added to $\frac{1}{15}$ of the mixture. Hence in all there will be added $\frac{56}{15} \times 15 = 56$ oz.

88. 5 lb. Troy $= 4.1\frac{1}{7}$ lb. Av. He bought the gold at \$192, and sold it at \$256 per lb. Hence he made \$64 per lb. Av. $\$64 \times 4.1\frac{1}{7} = \$263.31\frac{3}{7}$.

He made by selling by Av., since an Av. oz. is lighter than a Troy.

89. 412.5 gr. $\times$.193 = 79.6125 gr. $= \frac{79.6125}{15.432} = 5^{g}16-$.

90. 80 — 32 = 48. $\frac{5}{8}$ of 48 = $26\frac{2}{3}$. 98 — 32 = 66. $\frac{5}{8}$ of 66 = $36\frac{2}{3}$. $-28-32=-60$. $\frac{5}{8}$ of $-60=-33\frac{1}{3}$.

91. $57\times\frac{8}{5}=102.6$. $102.6+32=134.6$.

95. 20 less 4% of 20 = 19.2
1 — 5% of 1 = .95

960
1728
18.24
20
1.76 = $1\frac{19}{25}$.

96. The Germans paid $\frac{500}{88}=5\frac{15}{22}\%$; and the French, $\frac{300}{65\frac{1}{2}}=4\frac{76}{131}\%$.

If the debt ran only 10 yr., the Germans would pay 100 at compound interest at 5%, or 162.889 for 88; i.e., 1.85+ for 1. The French would pay the amount of 100 at 3% compound interest for 10 yr., or 134.3916 for $65\frac{1}{2}$; i.e., 2.05+ for 1.

97. $\frac{2\times 28000\times 12}{7912\times 5280}=\frac{35}{21758}$.

98. 32 dry quarts $=\frac{67.2}{57.75}\times 32$, or $\frac{6720\times 32}{5775}$ liquid quarts. This, at 15¢ per quart, is $\frac{6720\times 32\times 15}{5775}=558\frac{6}{11}$¢. Hence the gain was $558\frac{6}{11}-480=78\frac{6}{11}$, and the rate per cent $\frac{78\frac{6}{11}\times 100}{480}=16\frac{4}{11}\%$.

99. The amount of $500 at 10% simple interest for 4 yr., is $700.

At deferred annual interest the amount is $730, the $30

being the 6 yr. interest on \$50; one \$50 for 3 yr., one for 2, and one for 1.

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